MODELLING THE BEHAVIOR OF CONCRETE MEMBERS: DEVELOPMENTS SINCE THE COMPLETION OF EN 1998-3:2005

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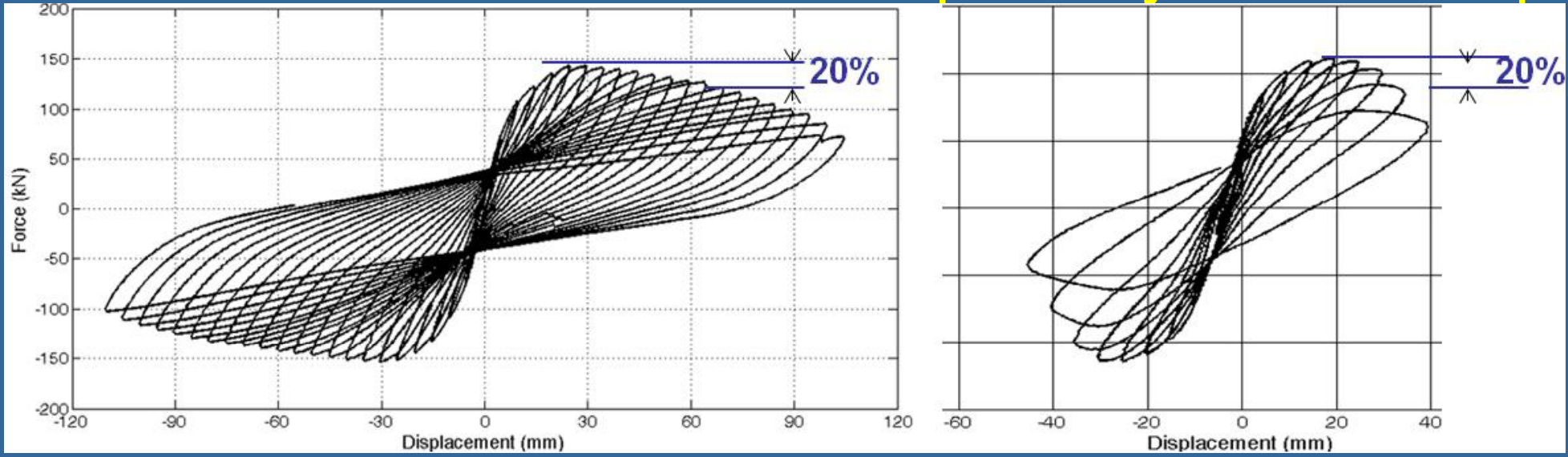
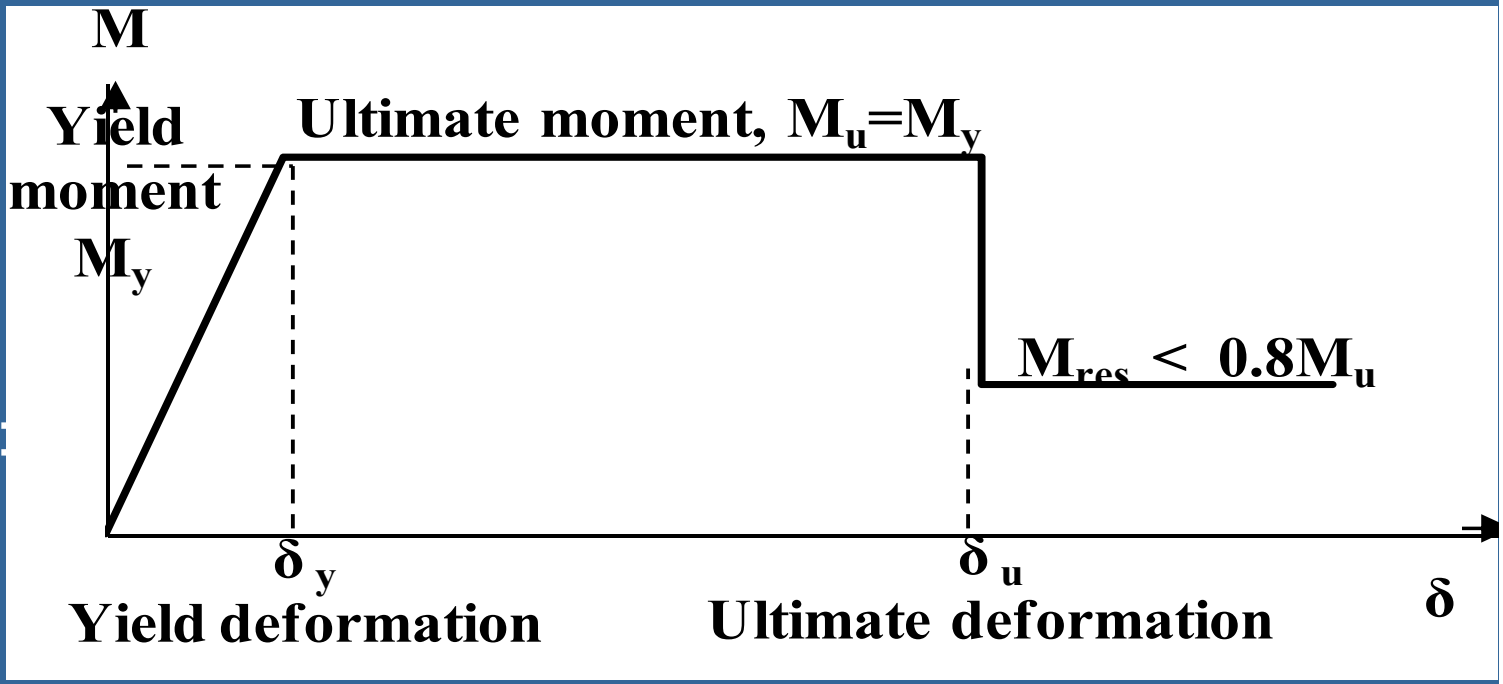
Idealized skeleton curve - envelope to hysteresis loops



Effective elastic stiffness: secant-to-yielding

member deformation: chord-rotation
section deformation: curvature



Practical expressions for the yield & failure properties developed for EN 1998-3:2005 - their advancement since 2005

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Experimental Database

1. Range/mean of parameters in tests for calibration of expressions for the member chord rotation and secant stiffness at yielding

Parameter	1653 rectangular beams/columns		214 rectangular walls		229 members of non-rectangular section		307 circular columns	
	min/max	mean	min/max	mean	min/max	mean	min/max	mean
section depth or diameter, h (m)	0.1 / 2.4	0.31	0.4 / 3.0	1.19	0.2 / 3.96	1.4	0.15 /1.83	0.43
shear-span-to-depth ratio, L_s/h	1 / 13.3	3.78	0.45 / 5.53	1.92	0.45 / 8.33	2.03	1.1 / 10	3.77
section aspect ratio, h/b_w	0.2 / 4	1.3	4 / 30	10.9	2.5 / 57	16.7	1.0	1.0
f_c (MPa)	9.6 / 175	37.7	13.5 / 109	35.8	13.5 / 101.8	40.4	16.7 / 90	36.7
axial-load-ratio, N/A_cf_c	-0.05 / 0.9	0.126	0 / 0.86	0.10	0 / 0.50	0.06	-0.1 / 0.7	0.137
transverse steel ratio, ρ_w (%)	0 / 3.54	0.62	0.05 / 2.18	0.54	0.04 / 2.44	0.7	0 / 8.83	0.86
total longitudinal steel ratio ρ_{tot} (%)	0.2 / 8.55	1.97	0.07 / 4.27	1.5	0.205 / 6.2	1.23	0.53 / 5.8	2.34
diagonal steel ratio, ρ_d (%)	0 / 1.68	0.027	0 / 0.25	0.005	-	-	-	-
transverse steel yield stress f_{yw}, MPa	118 / 2050	468	220 / 1375	443	160 / 1375	504	200 / 1728	454.2
longitudinal steel yield stress f_y, MPa	247 / 1200	440.2	276 / 1273	470	209 / 900	453	240 / 648	414.3

Experimental Database

2. Range/mean of parameters in tests for calibration of expressions for ultimate curvature in monotonic or cyclic loading

Parameter	415 rectangular beams/columns				59 rectangular walls			
	254 monotonic		160 cyclic		13 monotonic		46 cyclic	
	min/max	mean	min/max	mean	min/max	mean	min/max	mean
section depth or diameter, h (m)	0.12 / 0.8	0.31	0.22 / 2.4	0.41	2.39 / 2.41	2.4	1.7 / 2.0	1.99
section aspect ratio, h/b_w	0.225 / 3.73	1.54	0.5 / 2	1.21	21.2 / 23.4	22.4	13.3 / 28.3	13.7
f_c (MPa)	19.7 / 99.4	34.8	17.7 / 102.2	38.6	34.5 / 40.8	35.3	26.2 / 45.6	40.6
axial-load-ratio, $N/A_c f_c$	0 / 0.78	0.08	0 / 0.77	0.236	0.063 / 0.077	0.07	0.05 / 0.11	0.07
transverse steel ratio, ρ_w (%)	0 / 2.38	0.345	0.04 / 2.96	0.656	0.41 / 0.82	0.66	0.11 / 0.25	0.24
total longitudinal steel ratio ρ_{tot} (%)	0 / 3.68	1.4	0.37 / 4.19	1.82	0.88 / 1.76	1.35	0.07 / 0.77	0.63
transverse steel yield stress f_{yw}, MPa	0 / 596	419	255 / 1402	477.2	440 / 483	443.3	465 / 562	502
longitudinal steel yield stress f_y, MPa	277 / 596	490	341 / 573	493.3	444 / 510	466.6	523 / 580	552

Experimental Database

3. Range/mean of parameters in tests for calibration of expressions for the member ultimate cyclic chord rotation

Parameter	1159 rectangular beams/columns		95 rectangular walls		53 members of non-rectangular section		143 circular columns	
	min/max	mean	min/max	mean	min/max	mean	min/max	mean
section depth or diameter, h (m)	0.1 / 2.4	0.33	0.4 / 2.75	1.15	0.2 / 3.4	1.21	0.2 / 1.83	0.45
shear-span-to-depth ratio, L_s/h	1 / 13.3	3.7	0.5 / 5.53	2.15	0.65 / 8.33	2.85	1.77 / 10	4.22
section aspect ratio, h/b_w	0.2 / 6	1.18	2.5 / 28.3	9.75	2.5 / 36	9.95	1.0	1.0
f_c (MPa)	12.2 / 175	43.7	13.5 / 109	35.9	20.8 / 83.6	38.5	23.1 / 90	38.0
axial-load-ratio, $N/A_c f_c$	-0.1 / 0.9	0.165	0 / 0.86	0.116	0 / 0.30	0.07	-0.09 / 0.7	0.15
transverse steel ratio, ρ_w (%)	0.015 / 3.37	0.82	0.05 / 2.18	0.63	0.04 / 2.09	0.59	0.1 / 8.83	0.94
total longitudinal steel ratio ρ_{tot} (%)	0 / 6.29	2.08	0.07 / 4.27	1.37	0.2 / 6.19	1.32	0.75 / 5.5	2.05
diagonal steel ratio, ρ_d (%)	0 / 1.68	0.028	0 / 0.25	0.004	-	-	-	-
transverse steel yield stress f_{yw}, MPa	118 / 1497	497	220 / 1375	435.3	178 / 1375	513	200 / 1569	482.5
longitudinal steel yield stress f_y, MPa	281 / 1275	467.5	276 / 1273	471.2	331 / 596	438.7	240 / 648	428

Yield & failure properties of RC sections

M-φ at yielding of section w/ rectangular compression zone (width *b*, effective depth *d*) – section analysis

- Yield moment (from moment-equilibrium & elastic σ-ε laws):

$$\frac{M_y}{bd^3} = \varphi_y \left\{ E_c \frac{\xi_y^2}{2} \left(0.5(1+\delta_1) - \frac{\xi_y}{3} \right) + \frac{E_s}{2} \left[\left(1-\xi_y\right)\rho_1 + \left(\xi_y - \delta_1\right)\rho_2 + \frac{\rho_V}{6}\left(1-\delta_1\right) \right] \left(1-\delta_1\right) \right\}$$

- ρ_1, ρ_2 : tension & compression reinforcement ratios, ρ_v: "web" reinforcement ratio, ~uniformly distributed between ρ_1, ρ_2 : (all normalized to bd); $\delta_1 = d_1/d$.
- Curvature at yielding of tension steel:

$$\varphi_y = \frac{f_y}{E_s\left(1-\xi_y\right)d}$$

- from axial force-equilibrium & elastic σ-ε laws ($\alpha = E_s/E_c$):

$$\xi_y = \left(\alpha^2 A^2 + 2\alpha B\right)^{1/2} - \alpha A$$

$$A = \rho_1 + \rho_2 + \rho_v + \frac{N}{bdf_y}, \qquad B = \rho_1 + \rho_2\delta_1 + 0.5\rho_v\left(1+\delta_1\right) + \frac{N}{bdf_y}$$

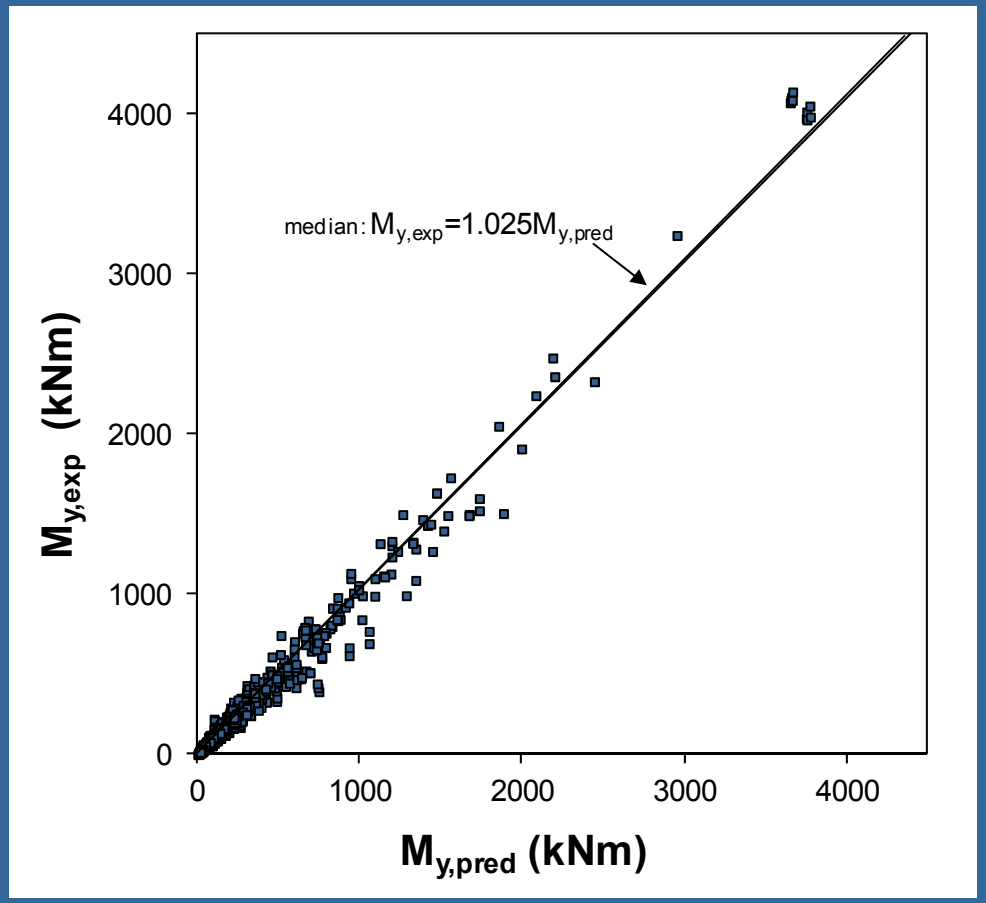
- Curvature at ~onset of nonlinearity of concrete:

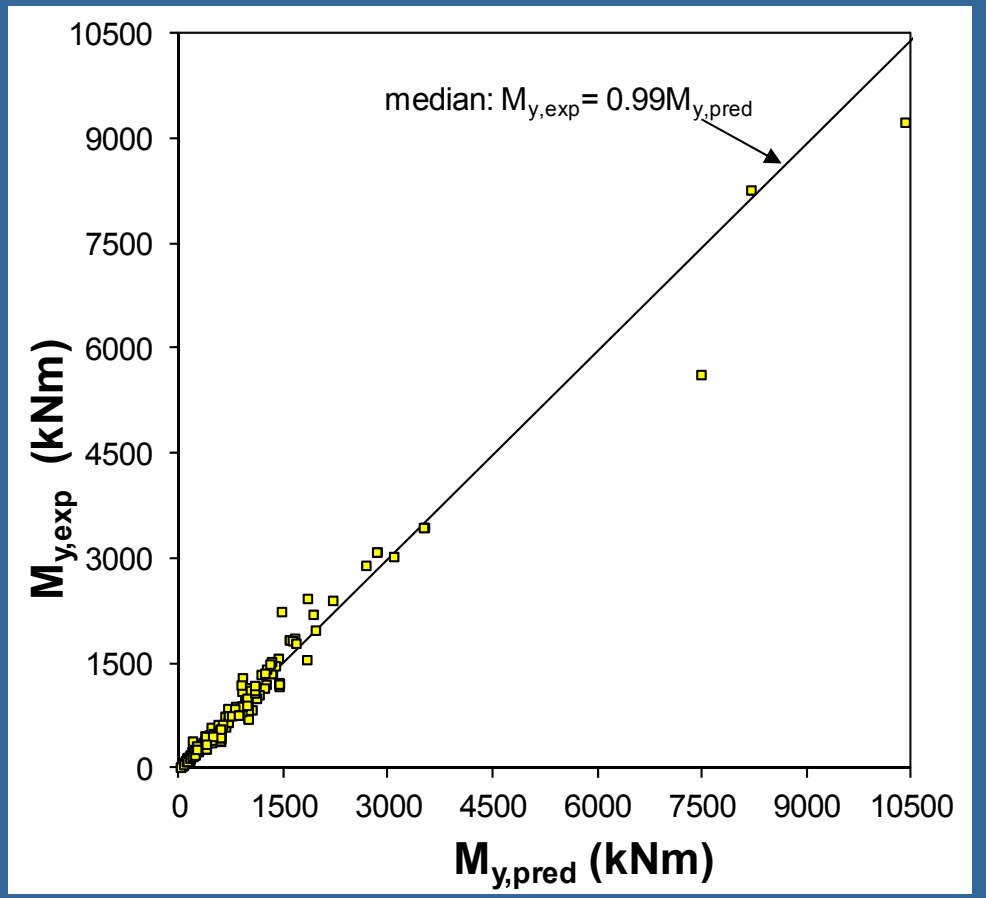
$$\varphi_y = \frac{\varepsilon_c}{\xi_y d} \approx \frac{1.8 f_c}{E_c \xi_y d}$$

$$A = \rho_1 + \rho_2 + \rho_v - \frac{N}{\varepsilon_c E_s bd} \approx \rho_1 + \rho_2 + \rho_v - \frac{N}{1.8\alpha\, bdf_c}, \quad B = \rho_1 + \rho_2\delta_1 + 0.5\rho_v\left(1+\delta_1\right)$$

Moment at corner of bilinear envelope to experimental moment-deformation curve vs yield moment from section analysis

Left: 2085 beams/columns, CoV:16.3%; Right: 224 rect. walls, CoV:16.9%





Bias by +2.5% or -1%, because corner of bilinear envelope of the experimental moment-deformation curve ≠ 1st yielding in section. Same bias considered to apply to predicted yield curvature.

Empirical formulas for yield curvature - section w/ rectangular compression zone

for beams or columns:

$$\varphi_y \approx \frac{1.54 f_y}{E_s d} \qquad \varphi_y \approx \frac{1.75 f_y}{E_s h}$$

for walls:

$$\varphi_y \approx \frac{1.37 f_y}{E_s d} \qquad \varphi_y \approx \frac{1.47 f_y}{E_s h}$$

Empirical expressions don't have a bias w.r.to experimental yield moment; but scatter is larger:

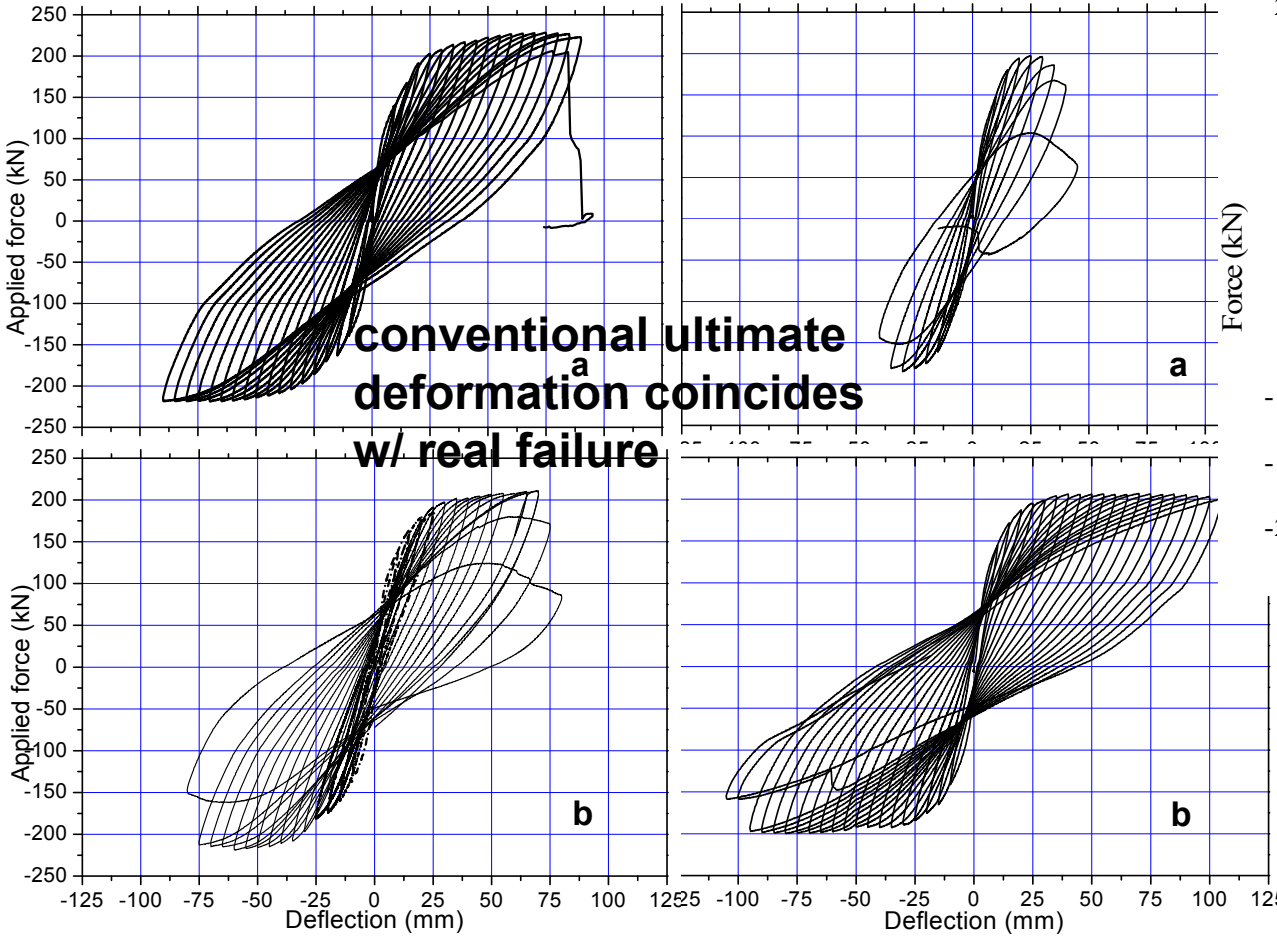
In ~2100 test beams, columns or walls: CoV: ~18%

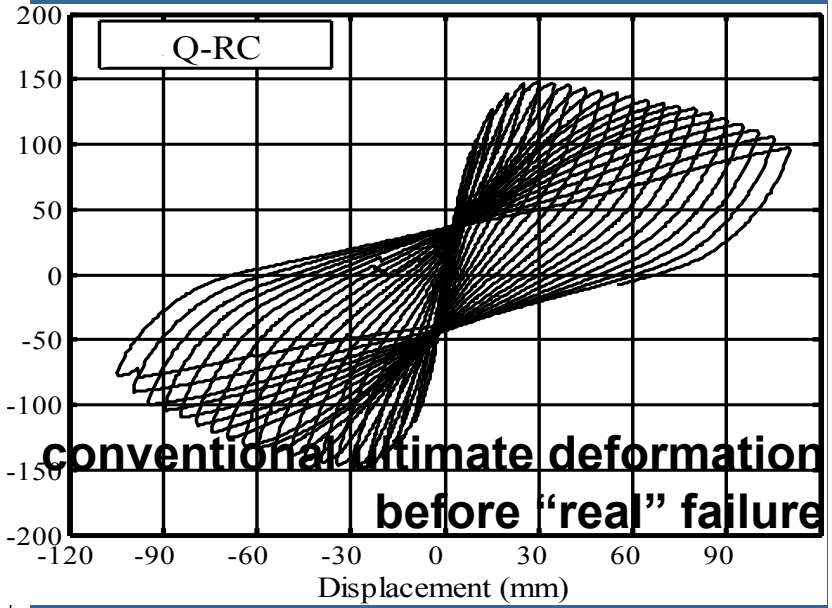
Flexural failures - columns

Flexural failures - beams

Conventional definition of ultimate deformation

The value beyond which, any increase in deformation cannot increase the resistance above 80% of the maximum previous (ultimate) resistance.





Ultimate curvature of section with rectangular compression zone, from section analysis

- Concrete σ-ε law:
  - Parabolic up to f_c, ε_{co},
  - constant stress (rectangular) for $\varepsilon_{co} < \varepsilon < \varepsilon_{cu}$
- Steel σ-ε law:
  - Elastic-perfectly plastic, if steel strain rather low and concrete fails first;
  - Elastic-linearly strain-hardening, if steel strains are relatively high and steel breaks at stress and strain f_t, ε_{su}.

Possibilities for ultimate curvature:

1. Section fails by rupture of tension steel, $\varepsilon_{s1} = \varepsilon_{su}$, before extreme compression fibers reach their ultimate strain (spalling), $\varepsilon_c < \varepsilon_{cu} \rightarrow$

   Ultimate curvature occurs in unspalled section, due to steel rupture: $\varphi_{su} = \dfrac{\varepsilon_{su}}{(1-\xi_{su})d}$

2. Compression fibres reach their ultimate strain (spalling): $\varepsilon_c = \varepsilon_{cu} \rightarrow$

   the confined concrete core becomes now the member section.

   Two possibilities:

   i. The moment capacity of the spalled section, M_{Ro}, never increases above 80% of the moment at spalling, M_{Rc}: $M_{Ro} < 0.8M_{Rc} \rightarrow$

      Ultimate curvature occurs in unspalled section, due to the concrete: $\varphi_{cu} = \dfrac{\varepsilon_{cu}}{\xi_{cu} d}$

   ii. Moment capacity of spalled section increases above

      80% of the moment at spalling: $M_{Ro} > 0.8M_{Rc} \rightarrow$

      The confined concrete core is now the member section and Cases 1 and 2(i) - applied for the confined core - are the two possibilities for attainment of the ultimate curvature $\rightarrow \varphi_{su}$, φ_{cu} calculated as above but for the confined core; the minimum of the two is the ultimate curvature.

$$M_{Rc} = bd^2 f_c \left\{ \frac{(1-\delta_1)(\omega_1+\omega_2)}{2} + \frac{\omega_v}{1-\delta_1} \left[(\xi-\delta_1)(1-\xi) - \frac{1}{3}\left(\frac{\xi f_y}{E_s \varepsilon_{cu}} \right)^2 \right] + \xi \left[\frac{1-\xi}{2} - \frac{\varepsilon_{co}}{3\varepsilon_{cu}} \left(\frac{1}{2} - \xi + \frac{\varepsilon_{co}}{4\varepsilon_{cu}} \xi \right) \right] \right\}$$

Ultimate curvature of section w/ rectangular compression zone for steel rupture:

$$\varphi_{su} = \frac{\varepsilon_{su}}{(1-\xi_{su})d}$$

- Steel ruptures before concrete crushes, after compression steel yields, if *v:*

$$\frac{\delta_1\varepsilon_{su}+\varepsilon_y-(1-\delta_1)\frac{\varepsilon_{co}}{3}}{\varepsilon_{su}+\varepsilon_y}+\omega_2-\omega_1\frac{f_t}{f_y}-\omega_v\left(1+\frac{f_t}{f_y}\right)\frac{\varepsilon_{su}-\varepsilon_y}{2(\varepsilon_{su}+\varepsilon_y)} \le \nu \le \frac{\varepsilon_{cu}-\frac{\varepsilon_{co}}{3}}{\varepsilon_{cu}+\varepsilon_{su}}+\omega_2-\omega_1\frac{f_t}{f_y}-\omega_v\left(1+\frac{f_t}{f_y}\right)\frac{\varepsilon_{su}(1+\delta_1)-\varepsilon_{cu}(1-\delta_1)}{(1-\delta_1)(\varepsilon_{su}+\varepsilon_{cu})}$$

- ξ_{su} calculated from axial force equilibrium for:

$$\xi_{su} = \frac{(1-\delta_1)\left(\nu+\omega_1\frac{f_t}{f_y}-\omega_2+\frac{\varepsilon_{co}}{3\varepsilon_{su}}\right)+\left(\frac{1+\delta_1}{2}\right)\left(1+\frac{f_t}{f_y}\right)\omega_v}{(1-\delta_1)\left(1+\frac{\varepsilon_{co}}{3\varepsilon_{su}}\right)+\left(1+\frac{f_t}{f_y}\right)\omega_v}$$

$\nu=N/bdf_c$, ω_1, ω_2 : tension & compression mech. reinforcement ratios, ωv: "web" mech. reinforcement ratio ~uniform distribution between ω_1, ω_2 ($\omega=\rho$ fy/fc); $\delta_1=d_1/d$.

- Steel ruptures before concrete crushes or compression steel yields, if *v:*

$$\nu \le \frac{\delta_1\varepsilon_{su}+\varepsilon_y-(1-\delta_1)\frac{\varepsilon_{co}}{3}}{\varepsilon_{su}+\varepsilon_y}+\omega_2-\omega_1\frac{f_t}{f_y}-\omega_v\left(1+\frac{f_t}{f_y}\right)\frac{\varepsilon_{su}-\varepsilon_y}{2(\varepsilon_{su}+\varepsilon_y)}$$

- ξ_{su} from: $\left[1+\frac{\varepsilon_{co}}{3\varepsilon_{su}}+\frac{\omega_v}{2(1-\delta_1)}\left(1+\frac{f_t}{f_y}-\frac{f_t}{E_s\varepsilon_{su}}-\frac{\varepsilon_{su}}{\varepsilon_y}\right)\right]\xi^2 -$

$$\left[1+\nu+\frac{2\varepsilon_{co}}{3\varepsilon_{su}}+\omega_1\frac{f_t}{f_y}+\omega_2\frac{\varepsilon_{su}}{\varepsilon_y}+\frac{\omega_v}{(1-\delta_1)}\left(1+\frac{f_t}{f_y}-\frac{f_t}{E_s\varepsilon_{su}}-\delta_1\frac{\varepsilon_{su}}{\varepsilon_y}\right)\right]\xi+\left[\nu+\frac{\varepsilon_{co}}{3\varepsilon_{su}}+\omega_1\frac{f_t}{f_y}+\omega_2\delta_1\frac{\varepsilon_{su}}{\varepsilon_y}+\frac{\omega_v}{2(1-\delta_1)}\left(1+\frac{f_t}{f_y}-\frac{f_t}{E_s\varepsilon_{su}}-\delta_1{}^2\frac{\varepsilon_{su}}{\varepsilon_y}\right)\right]=0$$

Ultimate curvature of section w/ rectangular compression zone for concrete crushing:

$$\varphi_{cu} = \frac{\varepsilon_{cu}}{\xi_{cu} d}$$

- Concrete crushes after tension steel yields, w/o compression steel yielding, if ν :

$$\nu \le \omega_2 - \omega_1 + \frac{\omega_v}{1-\delta_1}\left(\delta_1 \frac{\varepsilon_{cu}+\varepsilon_y}{\varepsilon_{cu}-\varepsilon_y} - 1\right) + \delta_1 \frac{\varepsilon_{cu} - \frac{\varepsilon_{co}}{3}}{\varepsilon_{cu}-\varepsilon_y}$$

- ξ_{cu} from axial force equilibrium, for:

$$\left[1 - \frac{\varepsilon_{co}}{3\varepsilon_{cu}} + \frac{\omega_v}{2(1-\delta_1)} \frac{(\varepsilon_{cu}+\varepsilon_y)^2}{\varepsilon_{cu}\varepsilon_y}\right]\xi^2 - \left[\nu + \omega_1 - \omega_2 \frac{\varepsilon_{cu}}{\varepsilon_y} + \frac{\omega_v}{1-\delta_1}\left(1 + \frac{\varepsilon_{cu}\delta_1}{\varepsilon_y}\right)\right]\xi - \left[\omega_2 - \frac{\omega_v \delta_1}{2(1-\delta_1)}\right]\frac{\varepsilon_{cu}\delta_1}{\varepsilon_y} = 0$$

- Concrete crushes w/ tension & compression steel yielding, if ν :

$$\omega_2 - \omega_1 + \frac{\omega_v}{1-\delta_1}\left(\delta_1 \frac{\varepsilon_{cu}+\varepsilon_y}{\varepsilon_{cu}-\varepsilon_y} - 1\right) + \delta_1 \frac{\varepsilon_{cu} - \frac{\varepsilon_{co}}{3}}{\varepsilon_{cu}-\varepsilon_y} \le \nu < \omega_2 - \omega_1 + \frac{\omega_v}{1-\delta_1}\left(\frac{\varepsilon_{cu}-\varepsilon_y}{\varepsilon_{cu}+\varepsilon_y} - \delta_1\right) + \frac{\varepsilon_{cu} - \frac{\varepsilon_{co}}{3}}{\varepsilon_{cu}+\varepsilon_y}$$

- ξ_{cu} from:

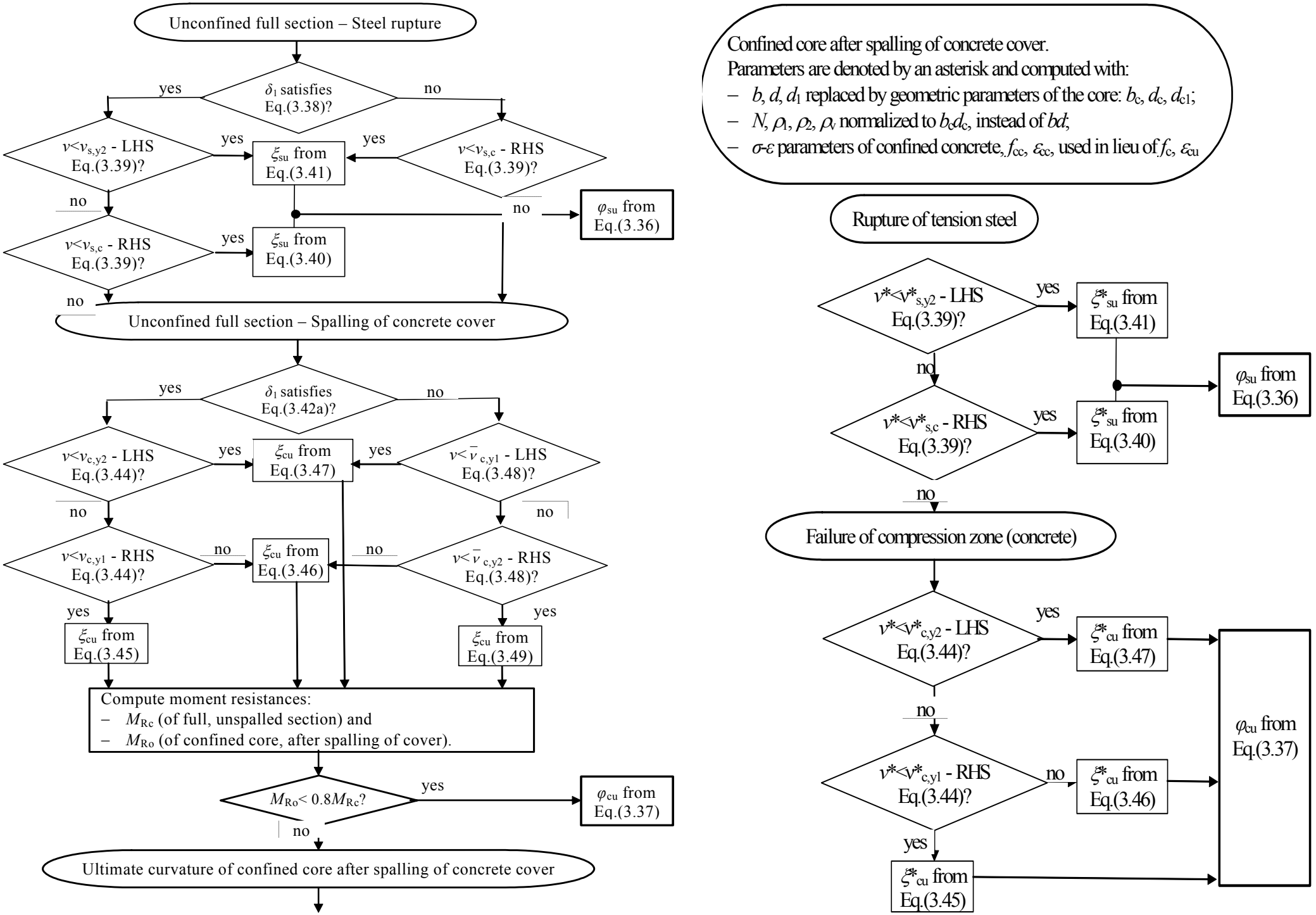
$$\xi_{cu} = \frac{(1-\delta_1)(\nu + \omega_1 - \omega_2) + (1+\delta_1)\omega_v}{(1-\delta_1)\left(1 - \frac{\varepsilon_{co}}{3\varepsilon_{cu}}\right) + 2\omega_v}$$

- Concrete crushes after compression steel yields, w/o tension steel yielding, if ν :

- ξ_{cu} from:

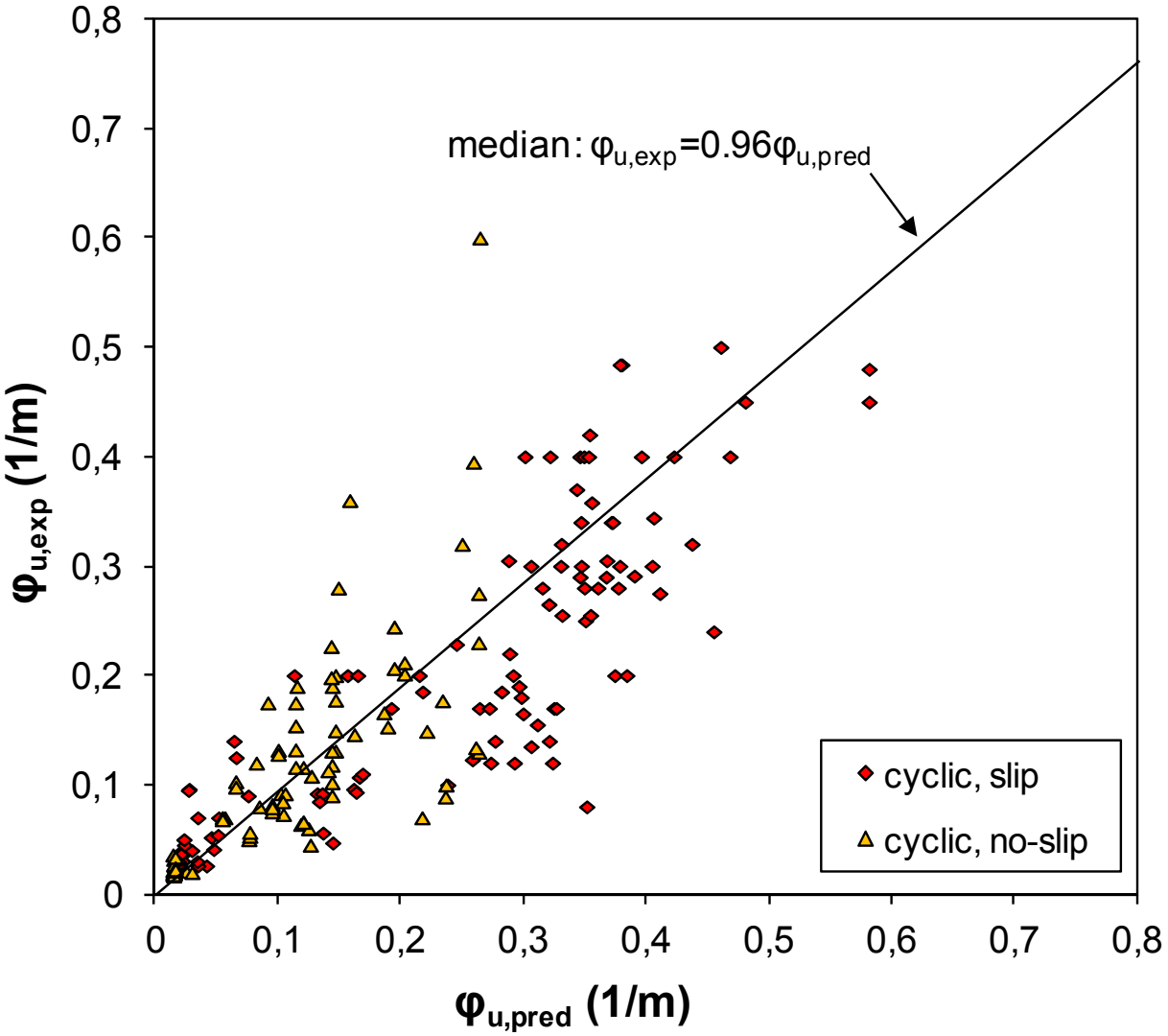
$$\omega_2 - \omega_1 + \frac{\omega_v}{1-\delta_1}\left(\frac{\varepsilon_{cu}-\varepsilon_y}{\varepsilon_{cu}+\varepsilon_y} - \delta_1\right) + \frac{\varepsilon_{cu} - \frac{\varepsilon_{co}}{3}}{\varepsilon_{cu}+\varepsilon_y} \le \nu$$

$$\left[1 - \frac{\varepsilon_{co}}{3\varepsilon_{cu}} - \frac{\omega_v}{2(1-\delta_1)} \frac{(\varepsilon_{cu}-\varepsilon_y)^2}{\varepsilon_{cu}\varepsilon_y}\right]\xi^2 + \left[\omega_2 + \omega_1 \frac{\varepsilon_{cu}}{\varepsilon_y} - \nu + \frac{\omega_v}{1-\delta_1}\left(\frac{\varepsilon_{cu}}{\varepsilon_y} - \delta_1\right)\right]\xi - \left[\omega_1 + \frac{\omega_v}{2(1-\delta_1)}\right]\frac{\varepsilon_{cu}}{\varepsilon_y} = 0$$


Unconfined full section – Steel rupture
δ_1 satisfies Eq.(3.38)?
yes
no
$\nu<\nu_{s,y2}$ - LHS Eq.(3.39)?
$\nu<\nu_{s,c}$ - RHS Eq.(3.39)?
ξ_{su} from Eq.(3.41)
ξ_{su} from Eq.(3.40)
φ_{su} from Eq.(3.36)
Unconfined full section – Spalling of concrete cover
δ_1 satisfies Eq.(3.42a)?
$\nu<\nu_{c,y2}$ - LHS Eq.(3.44)?
$\nu<\nu_{c,y1}$ - RHS Eq.(3.44)?
$\nu<\bar{\nu}_{c,y1}$ - LHS Eq.(3.48)?
$\nu<\bar{\nu}_{c,y2}$ - RHS Eq.(3.48)?
ξ_{cu} from Eq.(3.47)
ξ_{cu} from Eq.(3.46)
ξ_{cu} from Eq.(3.45)
ξ_{cu} from Eq.(3.49)
Compute moment resistances:
– M_{Rc} (of full, unspalled section) and
– M_{Ro} (of confined core, after spalling of cover).
$M_{Ro}< 0.8M_{Rc}$?
φ_{cu} from Eq.(3.37)
Ultimate curvature of confined core after spalling of concrete cover
Confined core after spalling of concrete cover.
Parameters are denoted by an asterisk and computed with:
– b, d, d_1 replaced by geometric parameters of the core: b_c, d_c, d_{c1};
– $N, \rho_1, \rho_2, \rho_v$ normalized to $b_c d_c$, instead of bd;
– σ-ε parameters of confined concrete, f_{cc}, ε_{cc}, used in lieu of f_c, ε_{cu}
Rupture of tension steel
$\nu^*<\nu^*_{s,y2}$ - LHS Eq.(3.39)?
$\nu^*<\nu^*_{s,c}$ - RHS Eq.(3.39)?
ξ^*_{su} from Eq.(3.41)
ξ^*_{su} from Eq.(3.40)
φ_{su} from Eq.(3.36)
Failure of compression zone (concrete)
$\nu^*<\nu^*_{c,y2}$ - LHS Eq.(3.44)?
$\nu^*<\nu^*_{c,y1}$ - RHS Eq.(3.44)?
ξ^*_{cu} from Eq.(3.47)
ξ^*_{cu} from Eq.(3.46)
ξ^*_{cu} from Eq.(3.45)
φ_{cu} from Eq.(3.37)

Test results vs ultimate curvature w/ failure strains for cyclic flexure per EN 1998-3:2005 :

Steel: ε_{su}: 2.5%, 5%, 6% for steel class A, B, C per Eurocode 2



$$\varepsilon_{cu,c} = 0.004 + 0.5\left(\alpha\rho_s f_{yw} / f_{cc}\right)$$

cyclic test results

	Cyclic all	Concrete crushing	Steel rupture
Median	0.96	0.90	0.98
C.o.V.	46.7%	55.1%	38.0%
No.	205	97	108

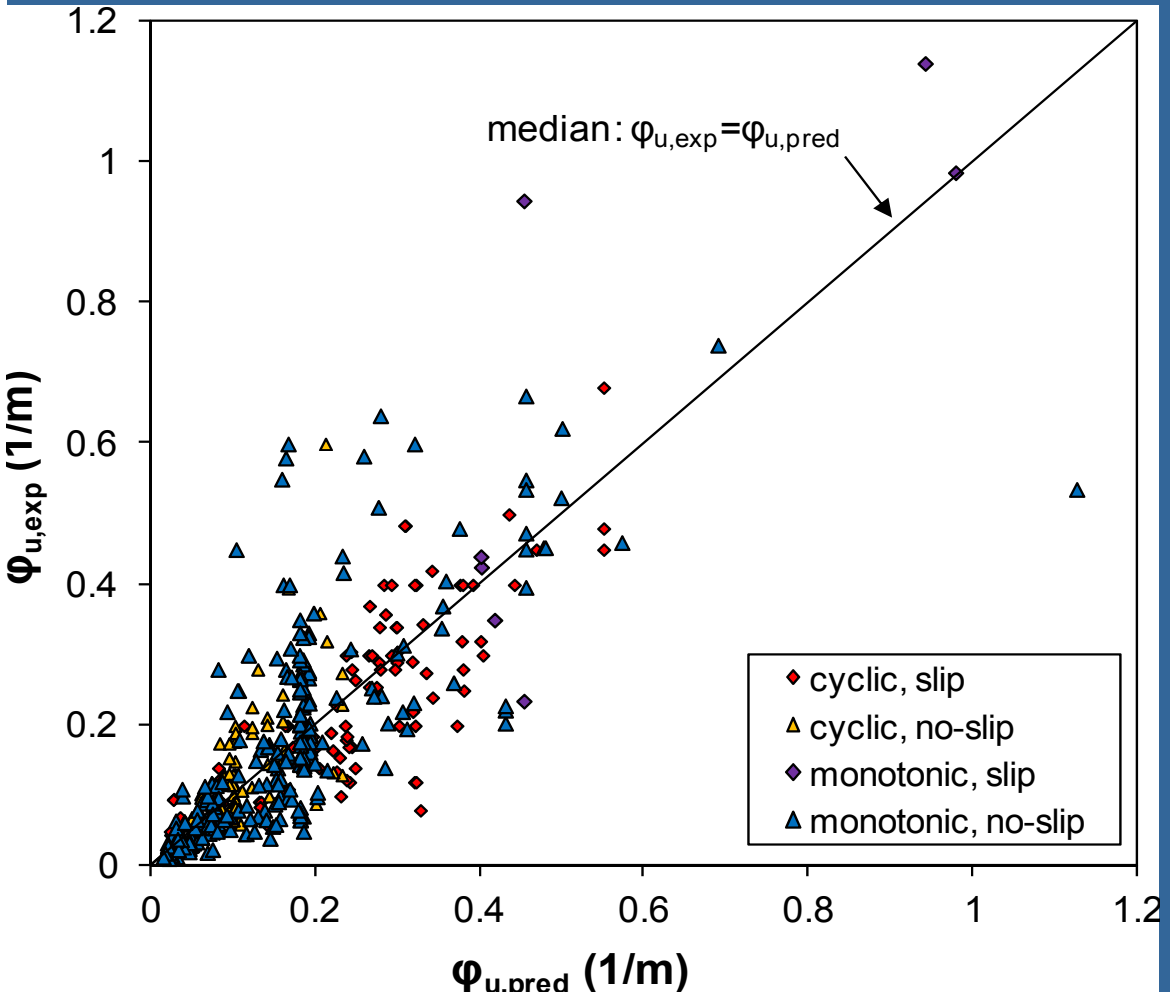
Test results vs ultimate curvature w/ failure strains for cyclic flexure per Biskinis & Fardis 2010 (adopted in *fib* MC2010):

- **Monotonic flexure:**
  - max. available steel strain: $(7/12)\varepsilon_{su}$

$$\varepsilon_{cu}^{*} = 0.0035 + \left(\frac{10}{h_c(mm)}\right)^2 + 0.285\frac{\alpha\omega_w}{1+K}$$

- **Cyclic flexure:**
  - max. available steel strain: $(3/8)\varepsilon_{su}$

$$\varepsilon_{cu}^{*} = 0.0035 + \left(\frac{10}{h_c(mm)}\right)^2 + 0.2\frac{\alpha\omega_w}{1+K}$$



monotonic & cyclic data, no. 474, median=1.00, CoV=49.7%

cyclic test data:

	Cyclic all	Concrete crushing	Steel rapture
Median	0.99	0.99	1.01
C.o.V.	44.2%	52.6%	34.2%
No.	205	97	108

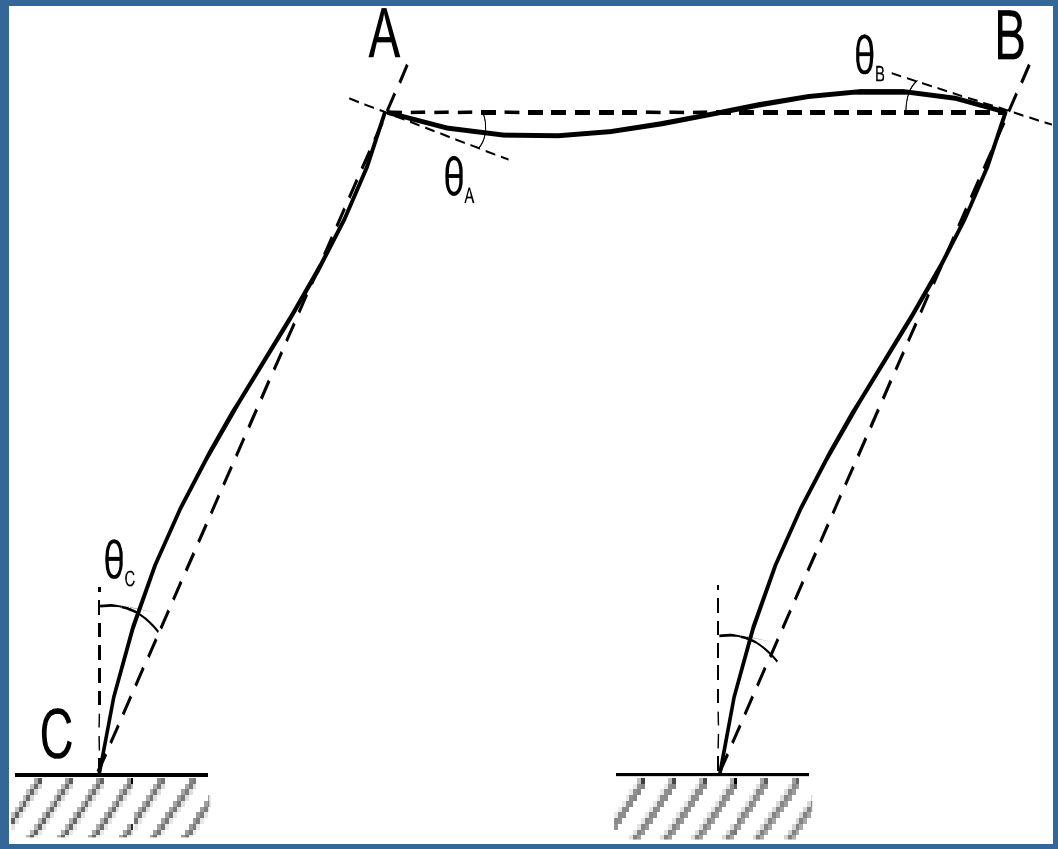
Yield & failure properties of RC members

Flexural behavior at member level (Moment-chord rotation)

Definition of chord rotations, θ, at member ends

$$\theta_A = \frac{1}{x_B - x_A} \int_{x_A}^{x_B} \varphi(x)(x_B - x)dx$$

$$\theta_B = \frac{1}{x_B - x_A} \int_{x_A}^{x_B} \varphi(x)(x_A - x)dx$$



Elastic moments at ends A, B from chord rotations at A, B:

- $M_A = (2EI/L)(2\theta_A+\theta_B)$,
- $M_B = (2EI/L)(2\theta_B+\theta_A)$

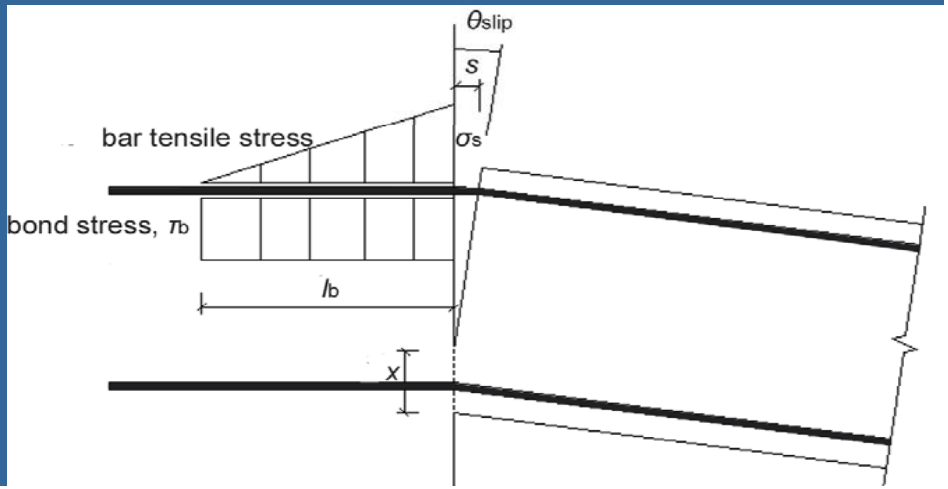
Fixed-end rotation of member end due to bar slippage from their anchorage zone beyond member end

- Slippage of tension bars from region beyond end section (e.g. from join or footing) →

  rigid-body rotation of entire shear span = fixed-end rotation, θ_{slip}

  (included in measured chord-rotations of test specimen w.r. to base or joint; doesn't affect measured relative rotations between any two member sections).
- If s = slippage of tension bars from anchorage → $\theta_{slip} = s/(1-\xi)d$
- If bond stress uniform over straight length l_b of bar beyond section of maximu moment → bar stress decreases along l_b from σ_s (=f_{yL} at yielding) at section o maximum M to zero at end of l_b → $s=\sigma_s l_b/(2E_s)$
- l_b= bond force demand per unit length (=$A_s\sigma_s/(\pi d_{bL})=d_{bL}\sigma_s/4$), divided by ~bon strength (assume =$\sqrt{f_c}$)
- $\varepsilon_s(=\sigma_s/E_s)/(1-\xi)d = \varphi$
- At yielding of member end section

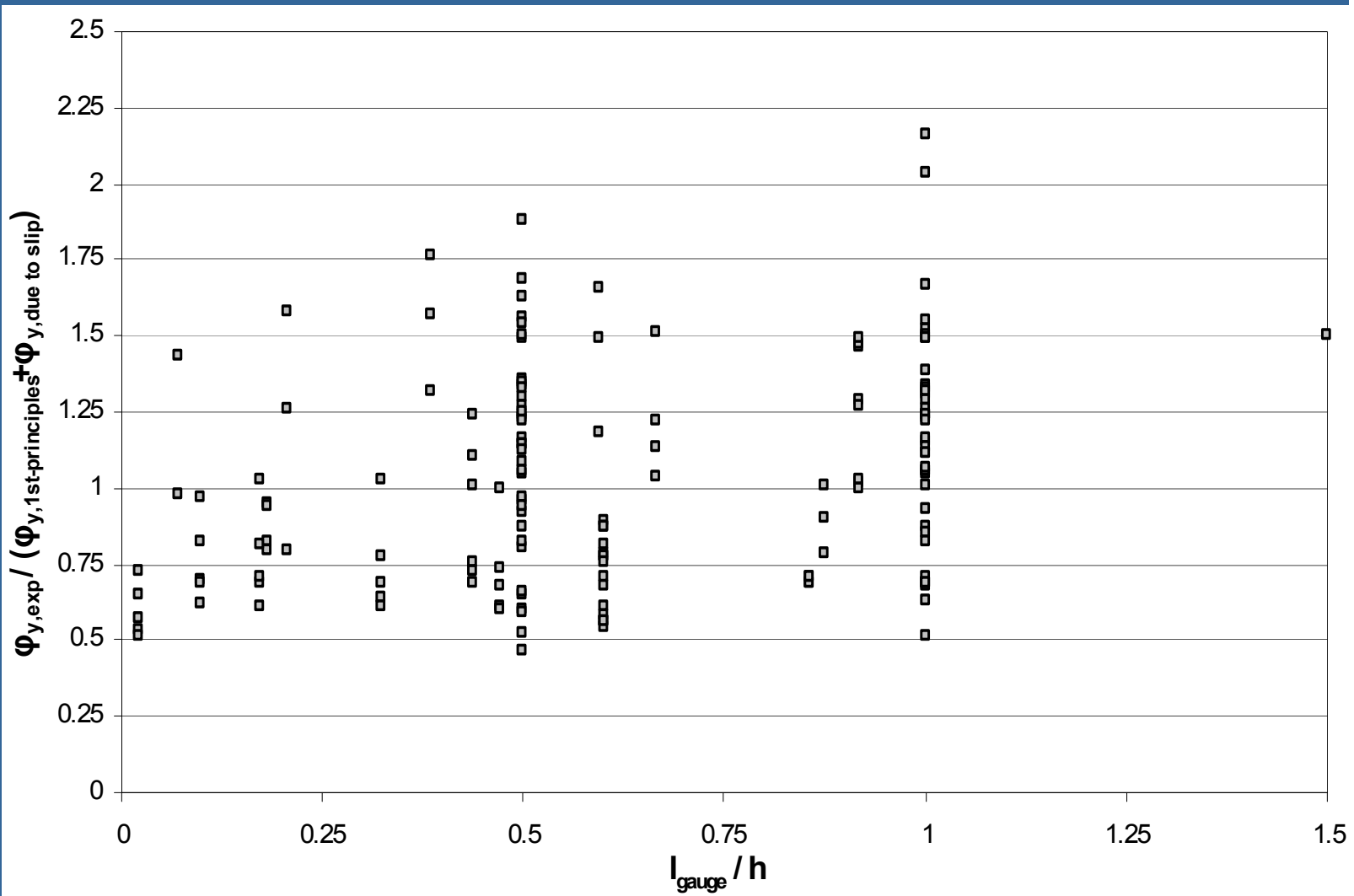
$$\theta_{y,slip} = \frac{\varphi_y d_{bL} f_y}{8\sqrt{f_c}}$$ (f_{yL}, f_c in MPa)



Fixed-end rotation of member end due to rebar pull-out from anchorage zone beyond member end, at member **yielding**

$\varphi_{y,measured}/(\varphi_{y,predicted}+\theta_{y,slip}/l_{gauge})$ no.160 measurements w/ slip: median = 1.0, C.o.V = 34%

Ratio: experimental-to-predicted yield curvature (w/ correction for fixed-end-rotation) in terms of gauge length



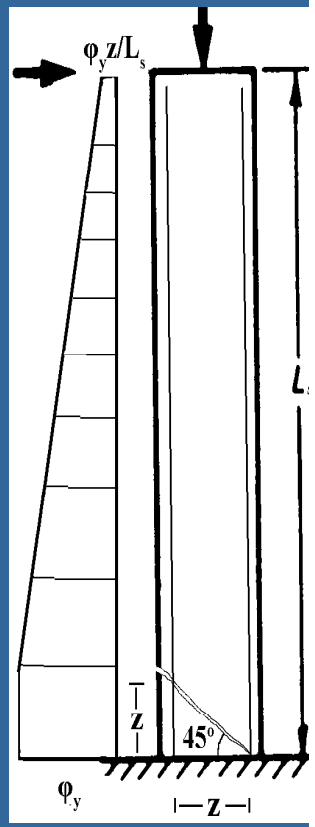
Chord rotation of shear span at yielding of end section per EN 1998-3:2005

- Rect. beams or columns:

$$\theta_y = \varphi_y \frac{L_s + a_V z}{3} + 0.0014\left(1 + 1.5\frac{h}{L_s}\right) + a_{sl}\theta_{y,slip}$$

- Rect. or non-rect. walls:

$$\theta_y = \varphi_y \frac{L_s + a_V z}{3} + 0.0013 + a_{sl}\theta_{y,slip}$$



"Shift rule":

Diagonal cracking shifts value of force in tension reinforcement to a section at a distance from member end equal to *z* (internal lever arm)

- $z = d - d_1$ in beams, columns, or walls of barbelled or T-section,
- $z = 0.8h$ in rectangular walls.

– $a_v = 0$, if $V_{Rc} > M_y/L_s$;

– $a_v = 1$, if $V_{Rc} \le M_y/L_s$.

V_{Rc} = force at diagonal cracking per Eurocode 2 (in kN, dimensions in m, f_c in MPa):

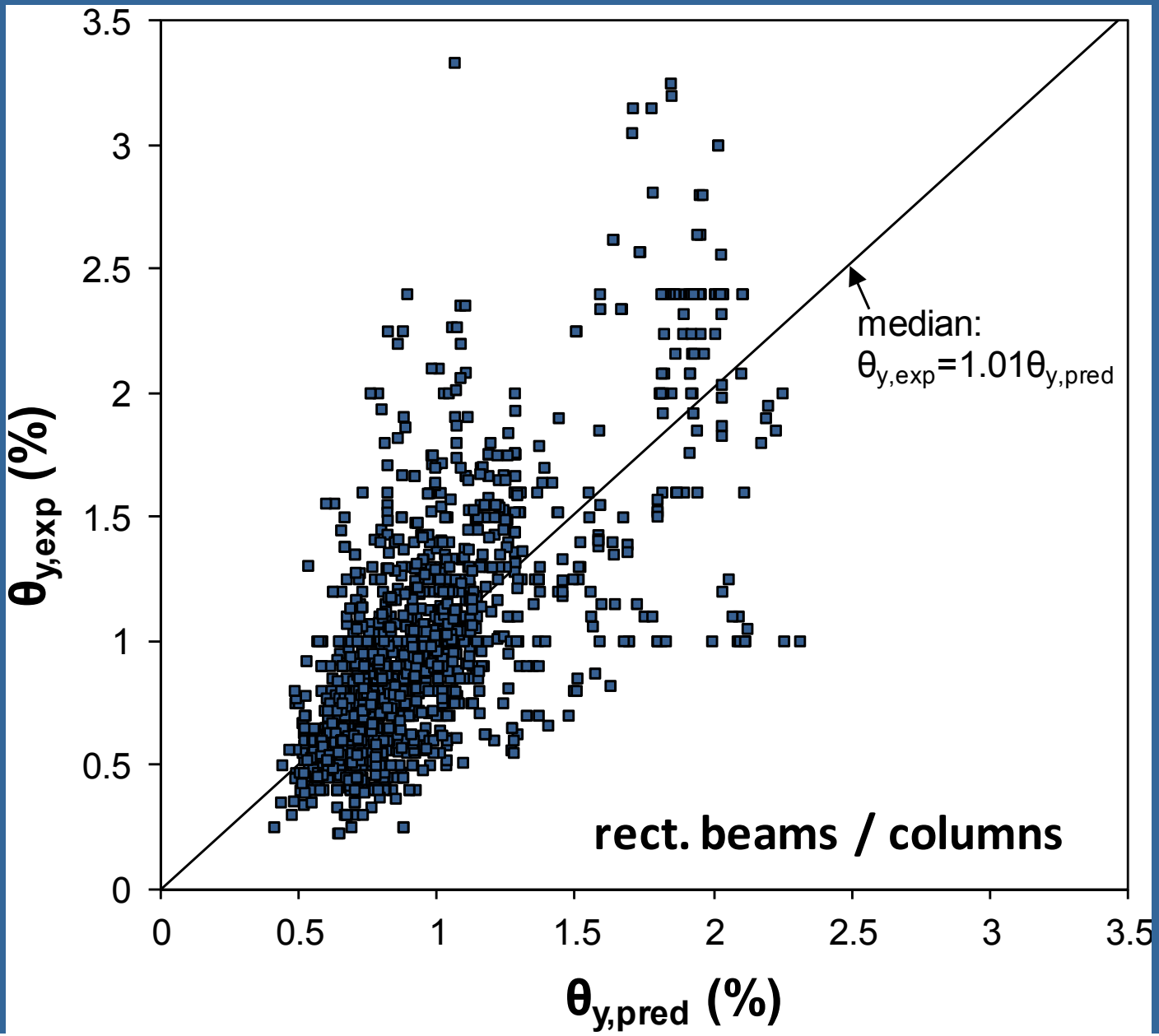
$$V_{R,c} = \left\{ \max\left[180(100\rho_1)^{1/3},\ 35\sqrt{1+\sqrt{\frac{0.2}{d}}}\, f_c^{1/6} \right] \left(1+\sqrt{\frac{0.2}{d}}\right) f_c^{1/3} + 0.15\frac{N}{A_c} \right\} b_w d$$

– $a_{sl} = 0$, if no slip from anchorage zone beyond end section;

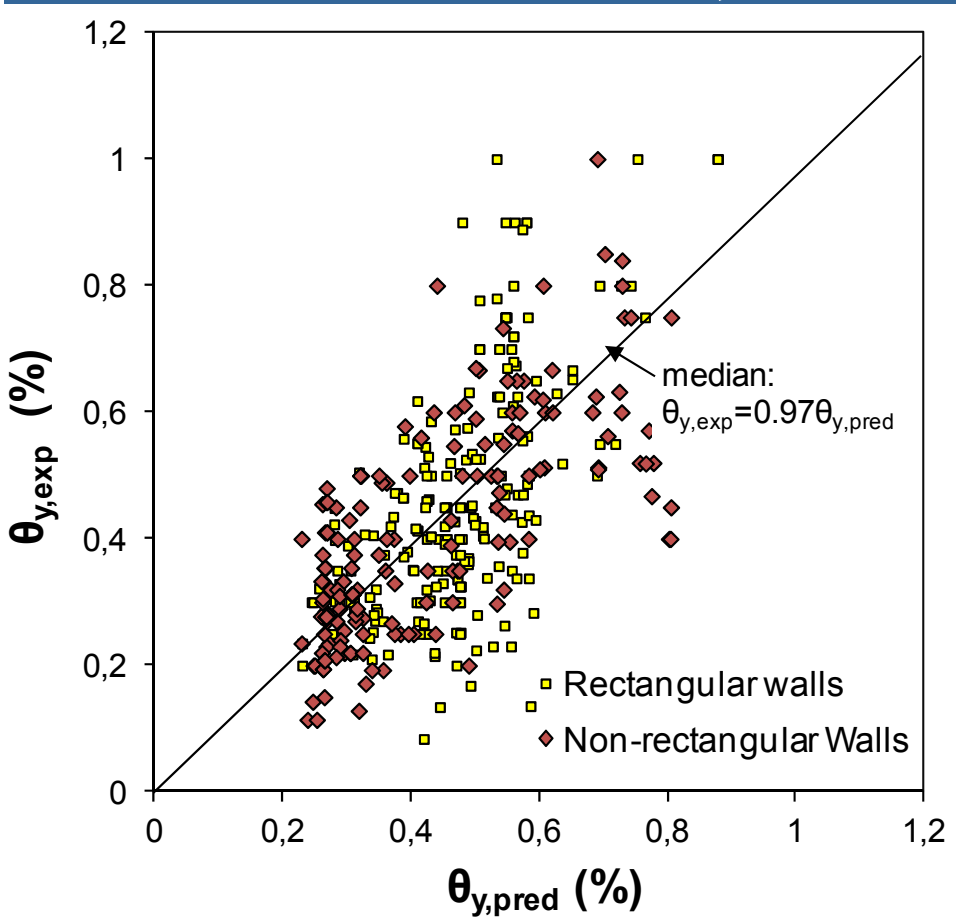
– $a_{sl} = 1$, if there is slip from anchorage zone beyond end section.

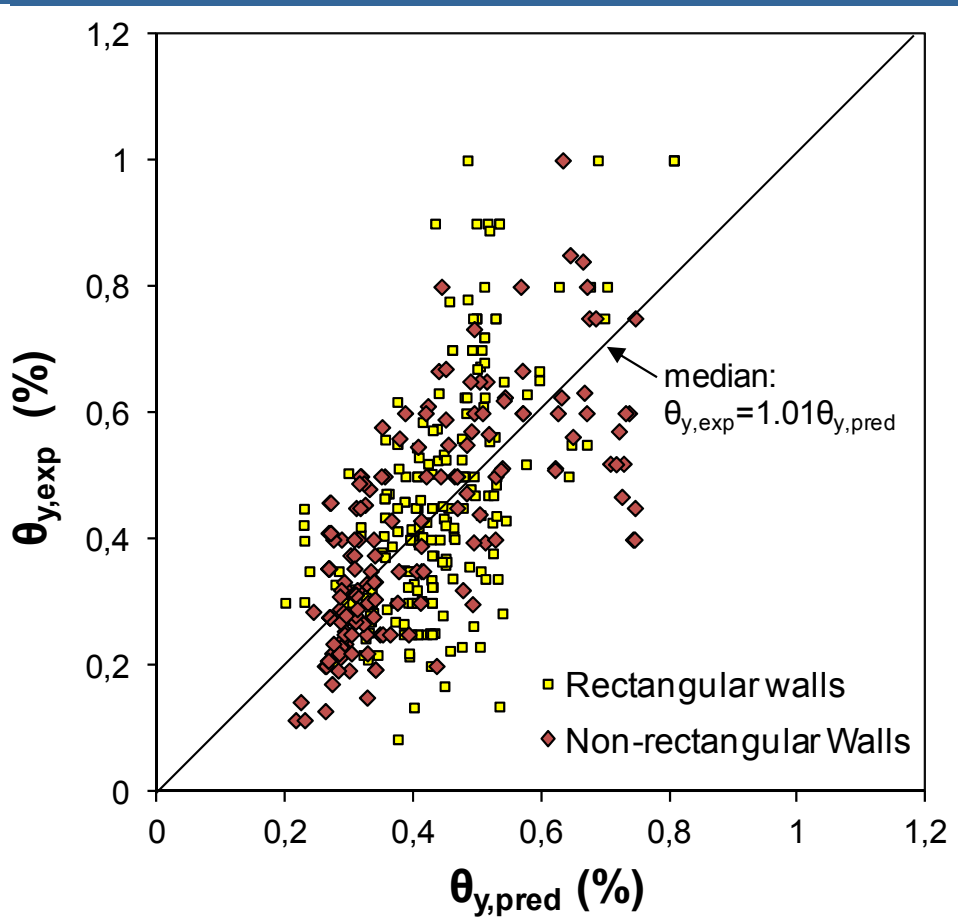
Test-model comparison – θ_y
Beams/rect. columns, no. tests: 1653

median=1.01,
CoV=32.1%



Test-model comparison – θ_y
Walls, no. tests: 386





Expression in EN 1998-3:2005
median=0.97, CoV=31.1%

Modified expression adopted in MC2010

$$\theta_y = \varphi_y \frac{L_s + a_V z}{3} + 0.00045\left(1 + \frac{5h}{3L_s}\right) + a_{sl}\theta_{y,slip}$$

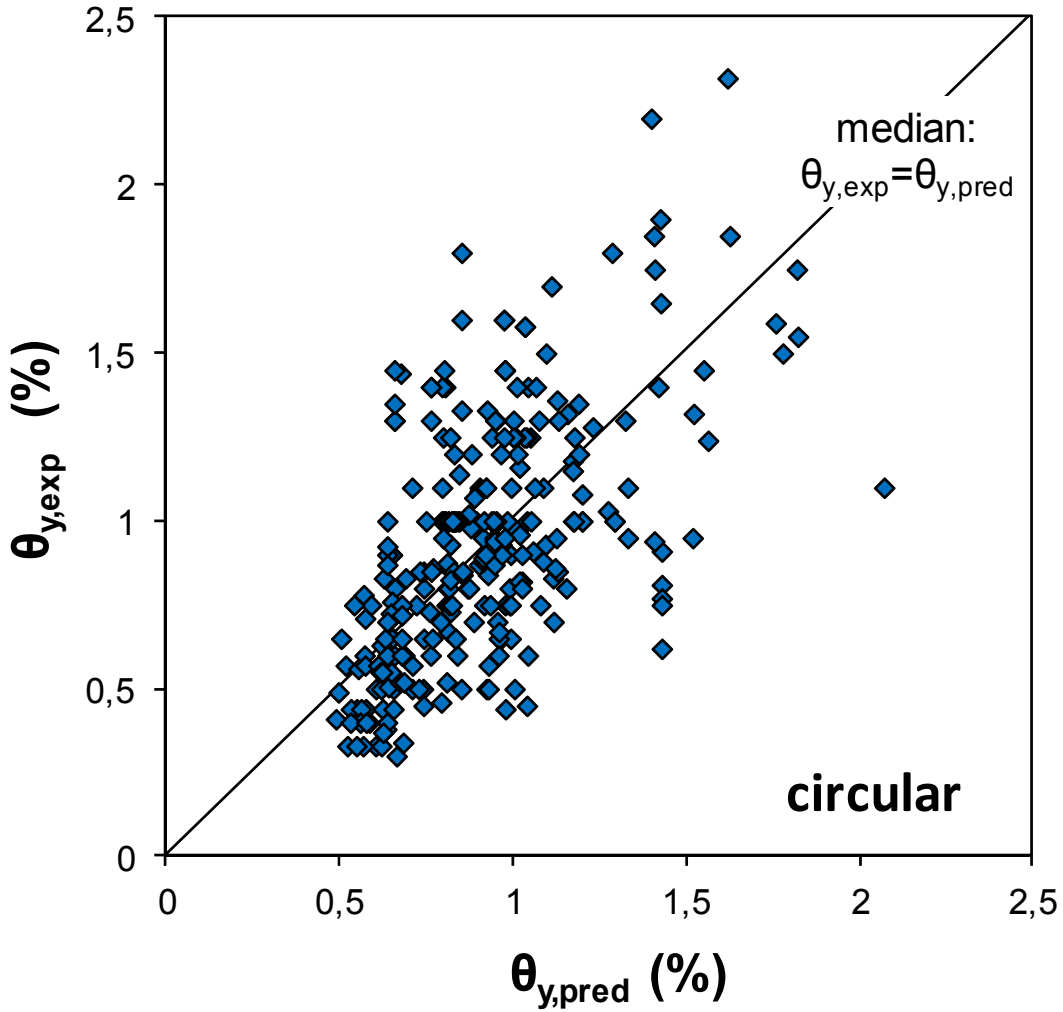
median=1.01, CoV=30.9%

Test-model comparisons - θ_y

Circular columns - not in EN 1998-3:2005: no. tests: 291

$$\theta_y = \varphi_y \frac{L_s + a_V z}{3} + 0.0027\left(1 - \min\left(1; \frac{2L_s}{15D}\right)\right) + a_{sl}\theta_{y,slip}$$

median=1.00, CoV=31.7%

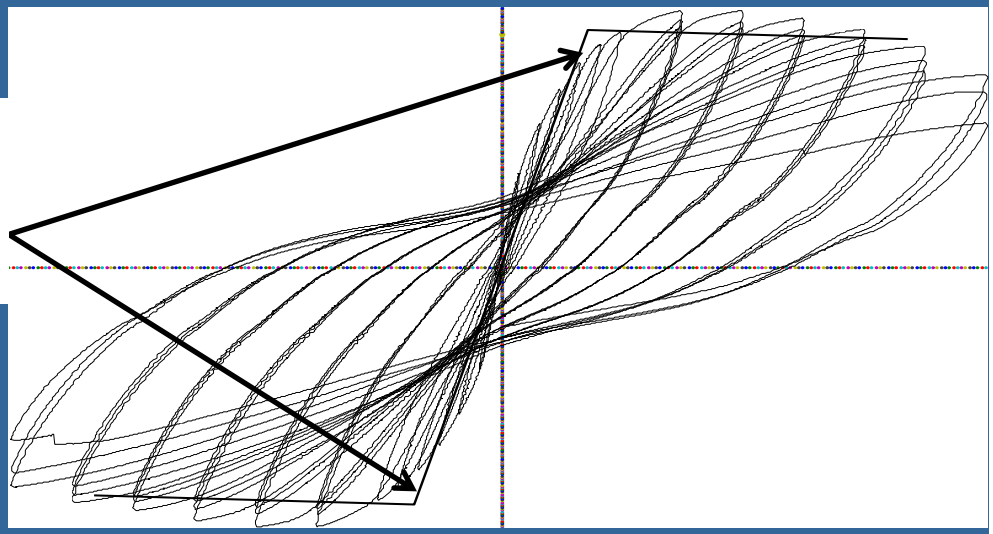


Effective elastic stiffness, EI_{eff} (for linear or nonlinear analysis)

- Part 1of EC8 (for design of new buildings):
  - EI_{eff} : secant stiffness at yielding =50% of uncracked gross-section stiffness.
  - OK in force-based design of new buildings (safe-sided for forces);
  - Unsafe in displacement-based assessment for displacement demands).
- More realistic:

$$EI_{eff} = \frac{M_y L_s}{3\theta_y}$$

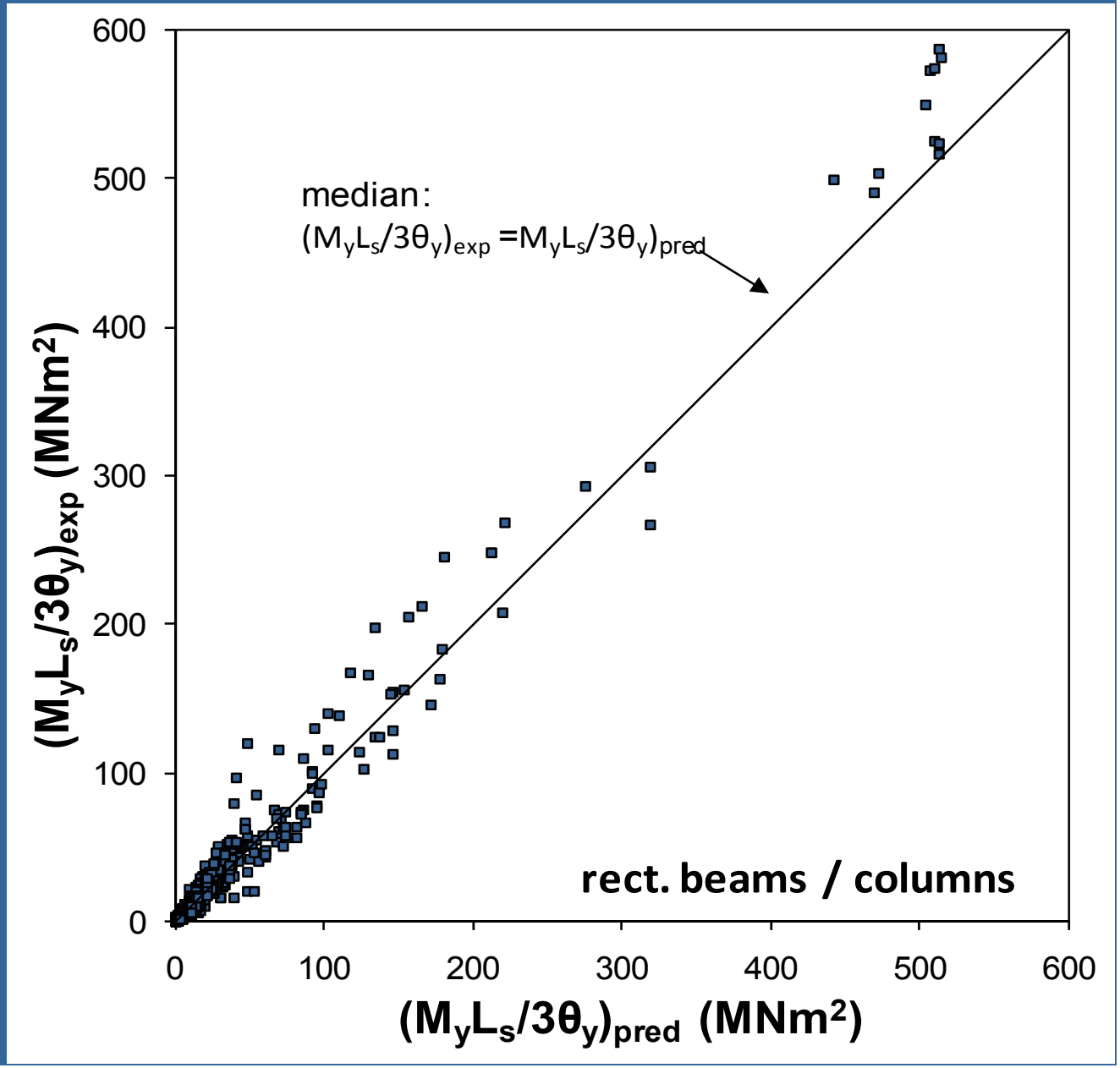
  - secant stiffness at yielding of end of shear span L_s=M/V



Test-model comparison – EI_{eff}
Beams/rect. columns, no. tests: 1616

median=1.00,
CoV=32.1%

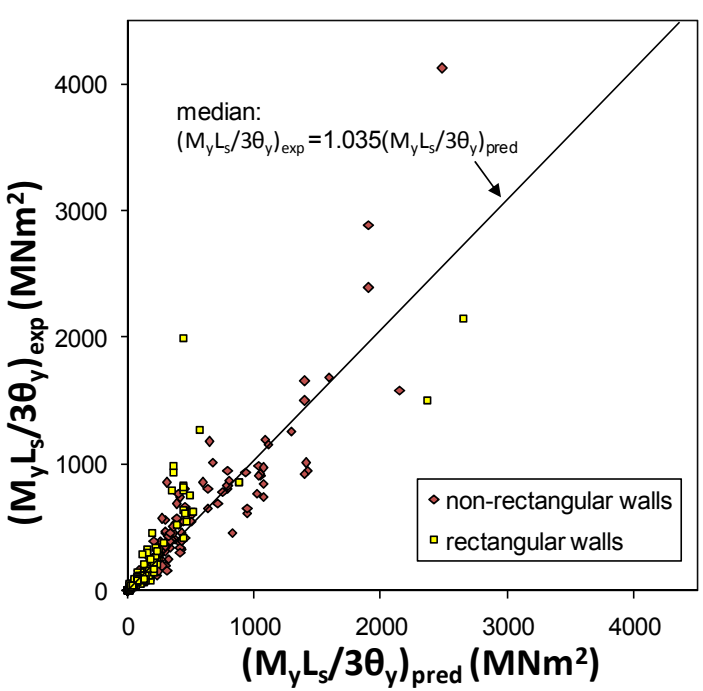
$$EI_{eff} = \frac{M_y L_s}{3\theta_y}$$

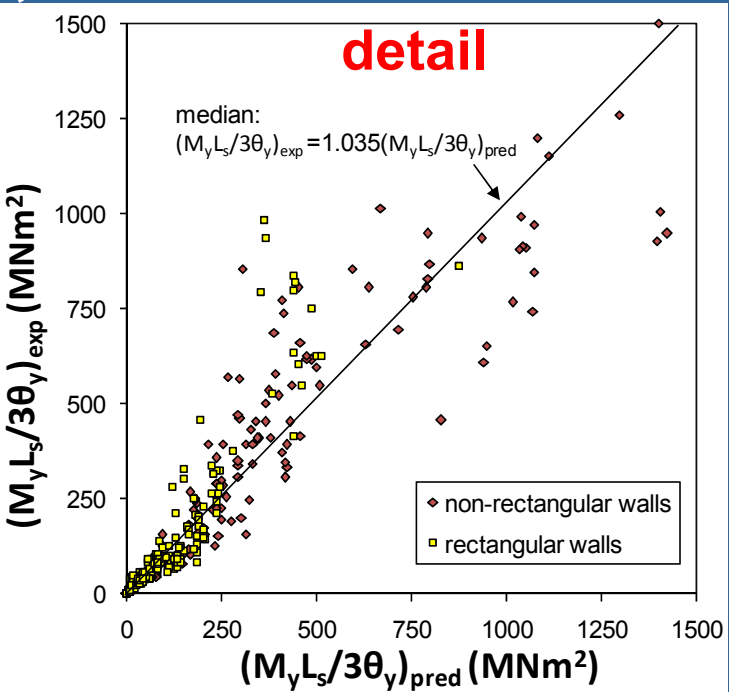


Test-model comparison – EI_{eff} Walls, no. tests: 386

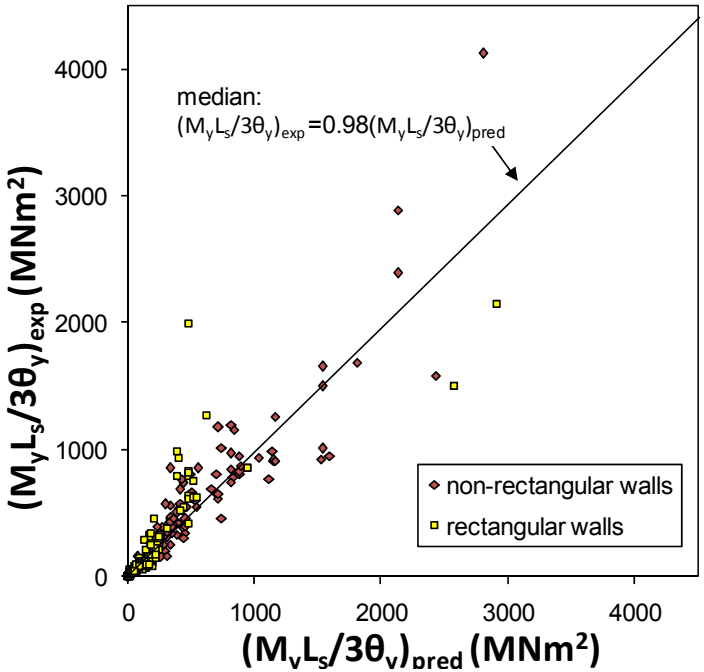
$$EI_{eff} = \frac{M_y L_s}{3\theta_y}$$

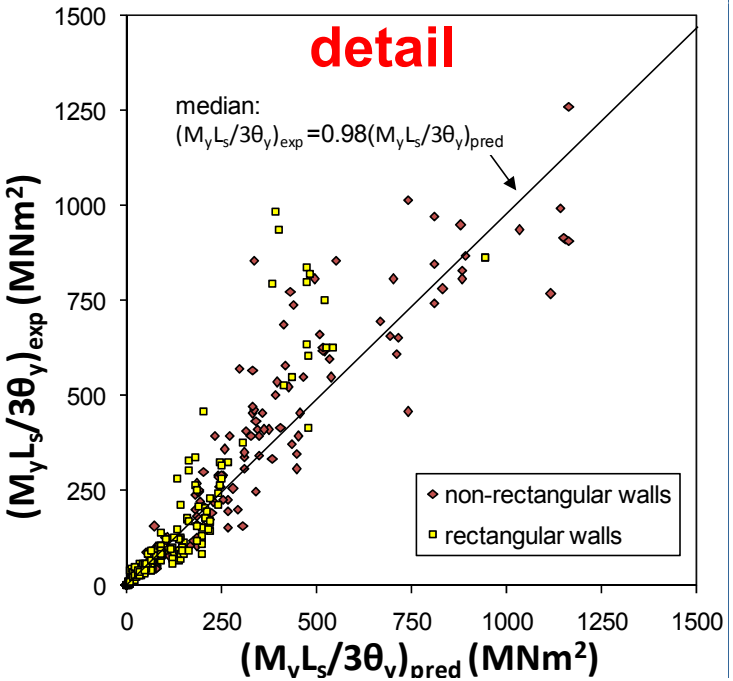
With expression for θ_y in EN 1998-3:2005
median=1.035
CoV=42.8%





With new expression for θ_y of walls (adopted in MC2010)
median=0.98
CoV=41.1%

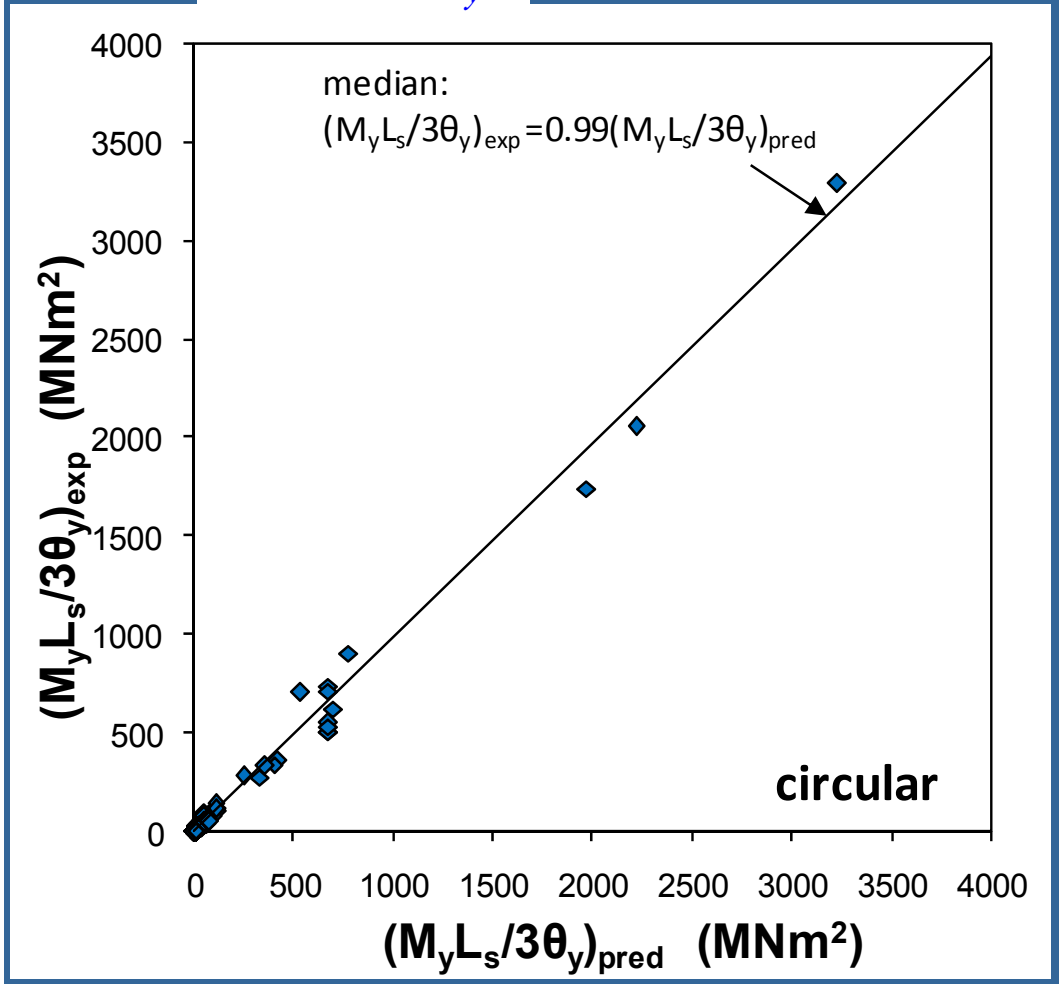


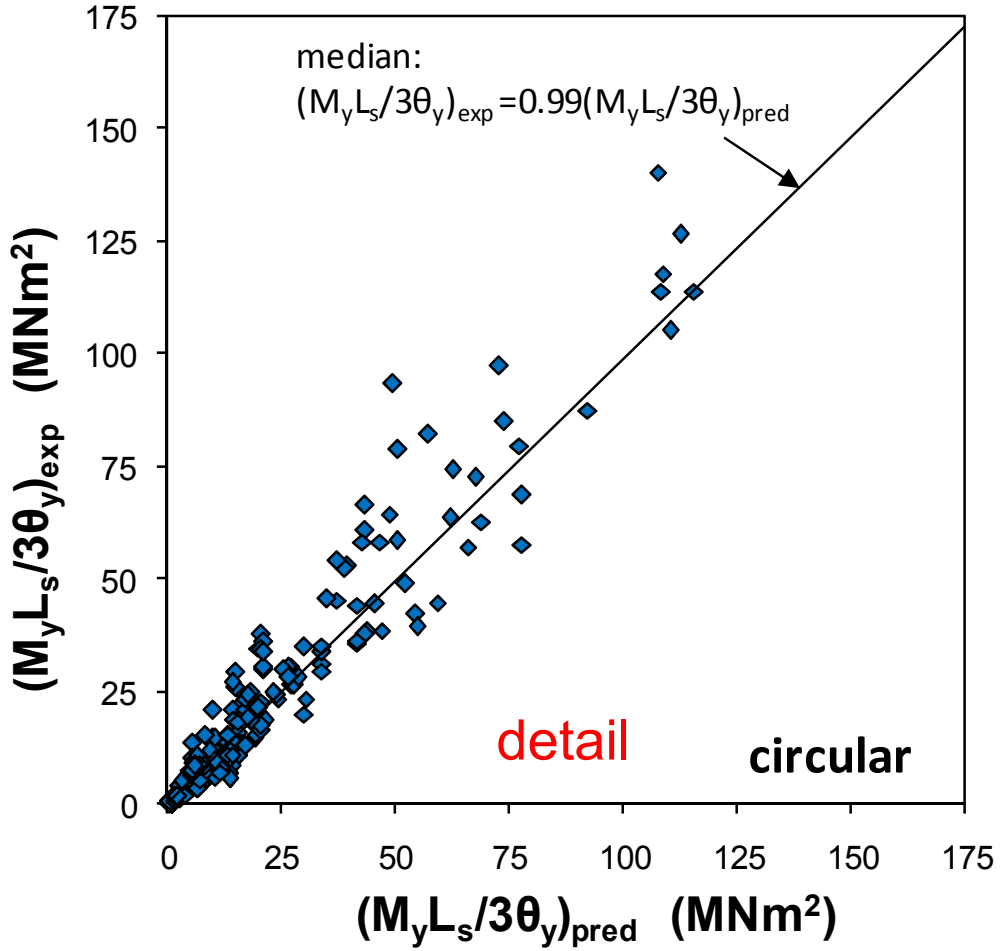


Test-model comparison – EI_{eff}
Circular columns - not in EN 1998-3:2005, no. tests: 273

$$EI_{eff} = \frac{M_y L_s}{3\theta_y}$$

median=0.99, CoV=31.2%





Empirical secant stiffness to yielding, EI_{eff}, independent of amount of reinforcement - not in EN 1998-3:2005

$$\frac{EI_{eff}}{E_c I_c} = a\left(0.8 + \ln\left(\max\left(\frac{L_s}{h};0.6\right)\right)\right)\left(1 + 0.048\min\left[50;\frac{N}{A_c}(MPa)\right]\right)$$

(all variables known before dimensioning the longitudinal reinforcement)

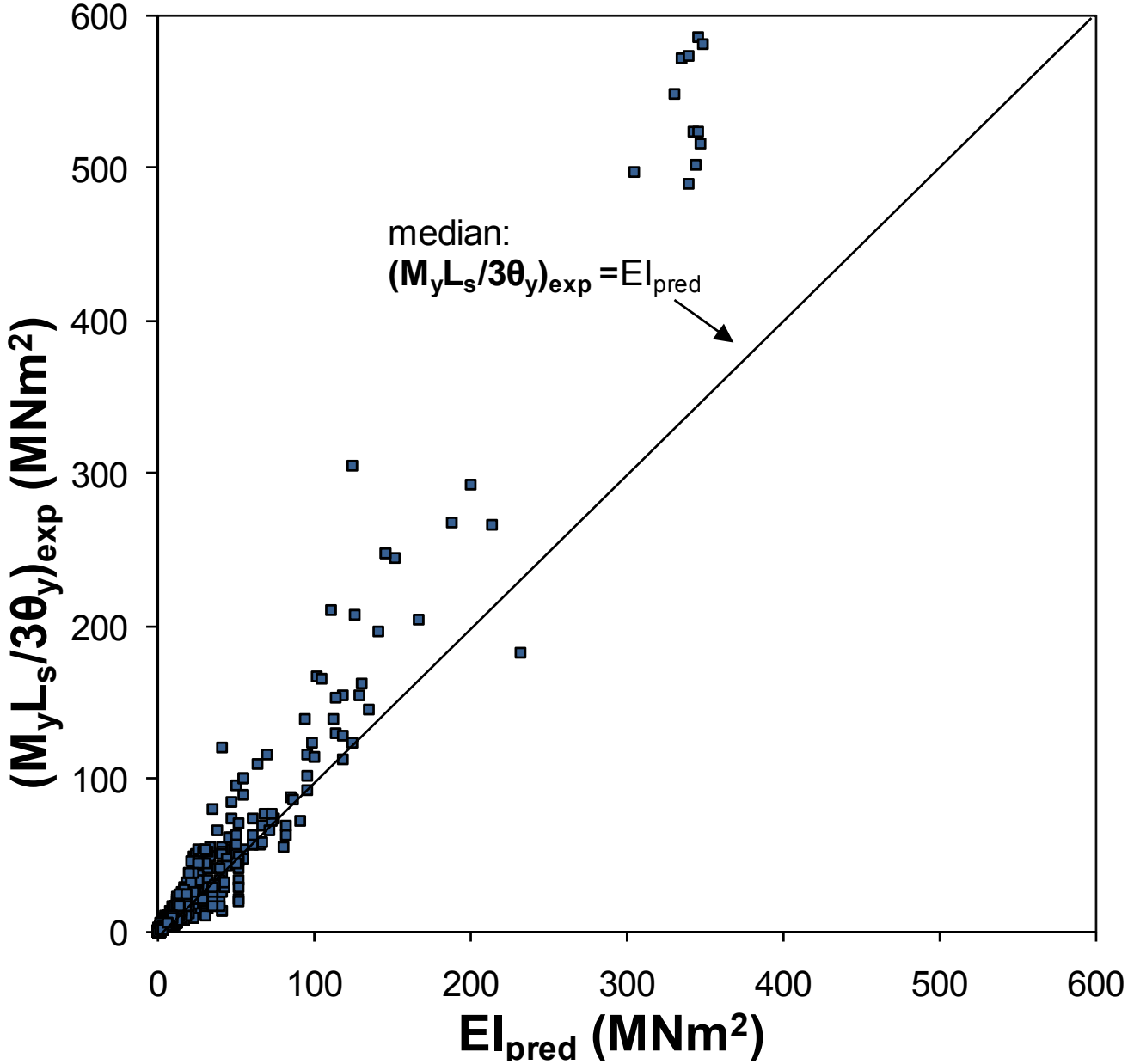
– If there is slippage of longitudinal bars from their anchorage zone beyond the member end:

- a = 0.081 for columns;
- a = 0.10 for beams or non-rectangular walls (barbelled, T-, H-section);
- a = 0.115 for rectangular walls;
- a = 0.12 for members with circular section.

– If there is no slippage of longitudinal bars: effective stiffness x 4/3

Test-model comparison – Empirical EI_{eff}, independent of amount of reinforcement - not in EN 1998-3:2005

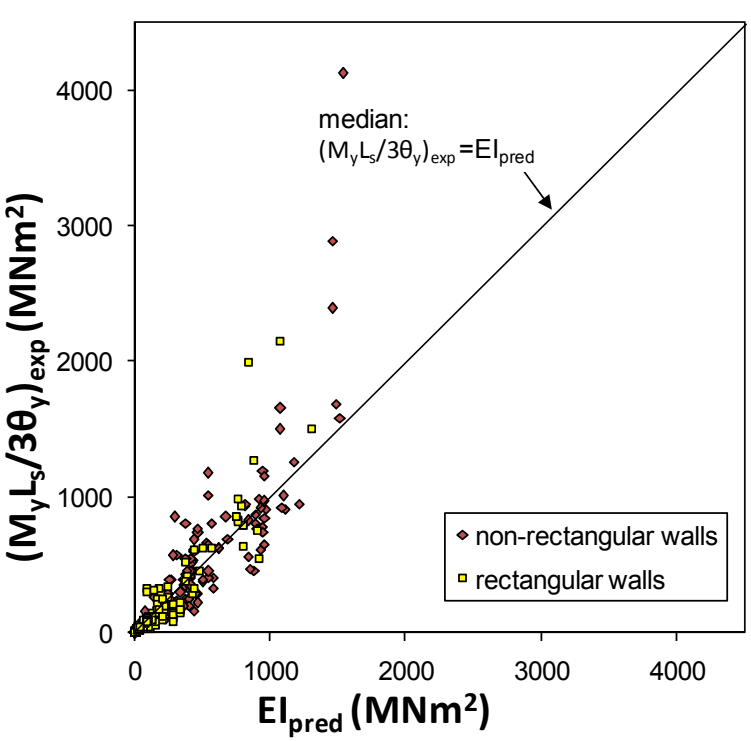
Beams/columns
no. tests: 1616
median=1.00
CoV=36.1%

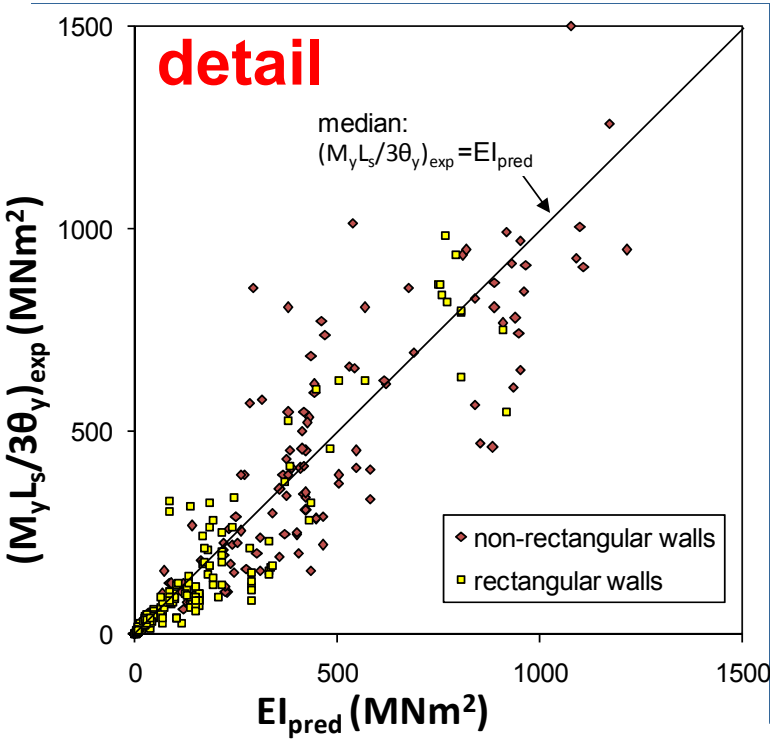


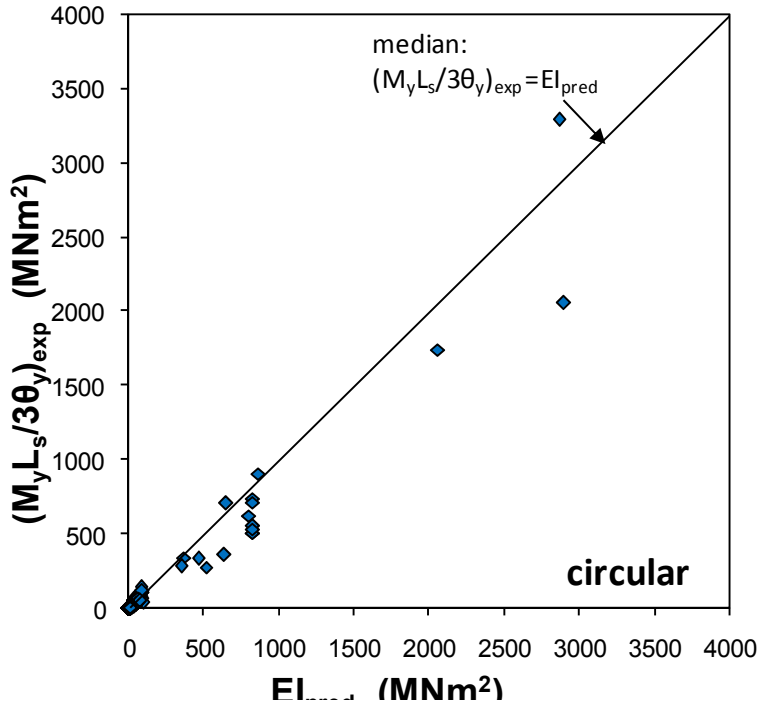
Test-model comparison – Empirical EI_{eff} independent of reinforcement , not in EN1998-3: 2005

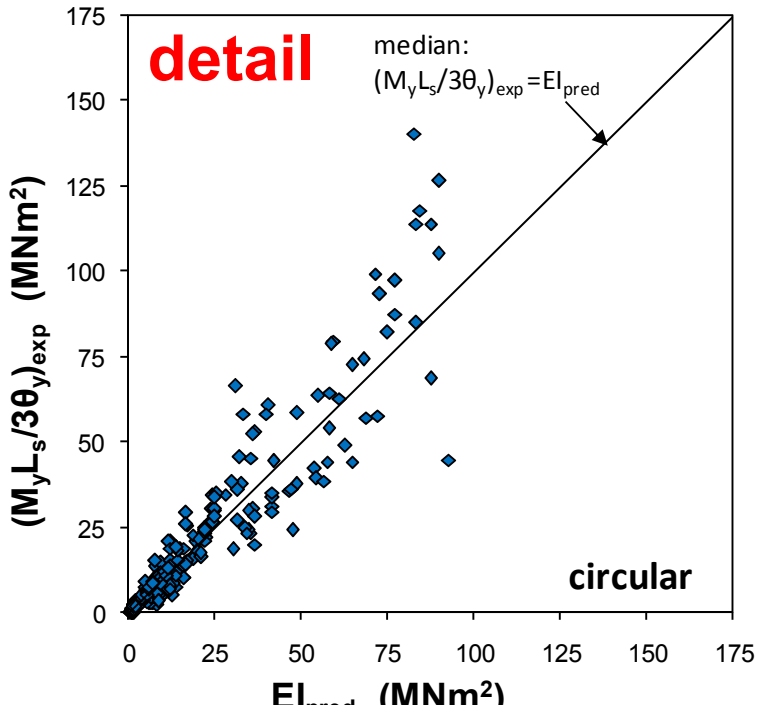
Walls
no. tests: 386
median=1.00
CoV=44.6%

Circular columns
no. tests: 273
median=0.995
CoV=31.4%









Flexure-controlled ultimate chord rotation from curvature & plastic hinge length per EN 1998-3:2005

$$\theta_u = \theta_y + \theta_u^{pl} = \theta_y + (\varphi_u - \varphi_y) L_{pl} \left(1 - \frac{0.5 L_{pl}}{L_s}\right)$$

- ϕ_y: yield curvature (section analysis);

$$\phi_u = \min\left(\frac{\varepsilon_{cu,c}}{\xi_{cu,c} d_c}, \frac{\varepsilon_{su}}{(1-\xi_{su}) d}\right)$$

Option 1: Confinement per Eurocode 2

$$\alpha\rho_{sx} f_{yw} \le 0.05 f_c: \quad f_{cc} = f_c + 5\alpha\rho_{sx} f_{yw}$$

$$\alpha\rho_{sx} f_{yw} > 0.05 f_c: \quad f_{cc} = 1.125 f_c + 2.5\alpha\rho_{sx} f_{yw}$$

$$\varepsilon_{cu,c} = \varepsilon_{co} + 0.2\left(\alpha\rho_{sx} f_{yw} / f_c\right)$$

$$L_{\mathrm{pl}} = 0.1 L_s + 0.17 h + 0.24 d_b f_y \Big/ \sqrt{f_c}$$

Option 2: New, per EN 1998-3:2005

$$f_{cc} = f_c \left[1 + 3.7\left(\alpha\rho_{sx} f_{yw} / f_c\right)^{0.86}\right]$$

$$\varepsilon_{cu,c} = 0.004 + 0.5\left(\alpha\rho_{sx} f_{yw} / f_{cc}\right)$$

$$L_{\mathrm{pl}} = L_s / 30 + 0.2 h + 0.11 d_b f_y / \sqrt{f_c}$$

index c: confined;

ρ_s: stirrup ratio;

L_s=M/V: shear span at member end;

h: section depth;

d_b: bar diameter;

f_y, f_c: MPa

α: confinement effectiveness:

– rectangular section:

$$\alpha = \left(1 - \frac{s_h}{2b_c}\right)\left(1 - \frac{s_h}{2h_c}\right)\left(1 - \frac{\sum b_i^2}{6 b_c h_c}\right)$$

– circular section & hoops:

$$\alpha = \left(1 - \frac{s_h}{2D_c}\right)^2$$

s_h: centerline spacing of stirrups,

D_c, b_c, h_c: confined core dimensions to centerline of hoop;

b_i: centerline spacing along section perimeter of longitudinal bars (index: i) engaged by a stirrup corner or cross-tie.

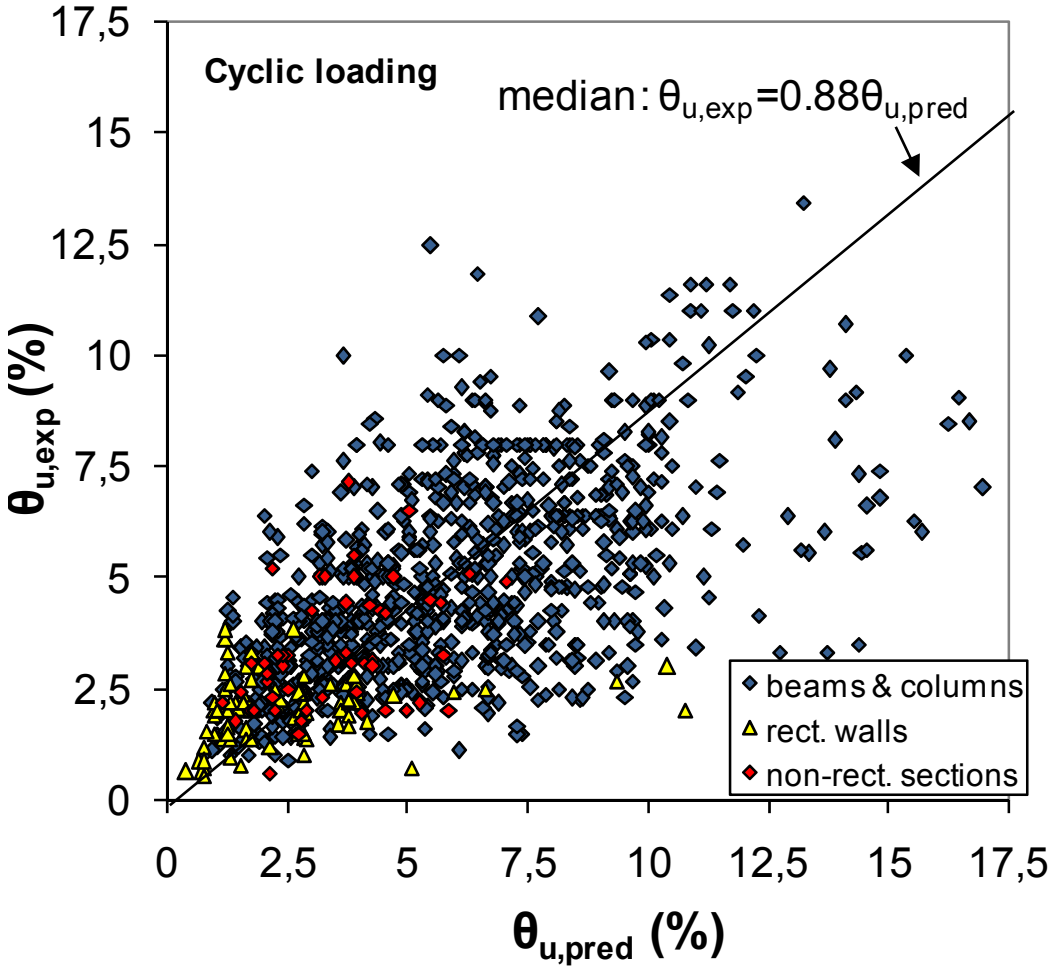
Test-model comparison – ultimate chord rotation from curvatures & plastic hinge length per EN 1998-3:2005, no. tests: 1100

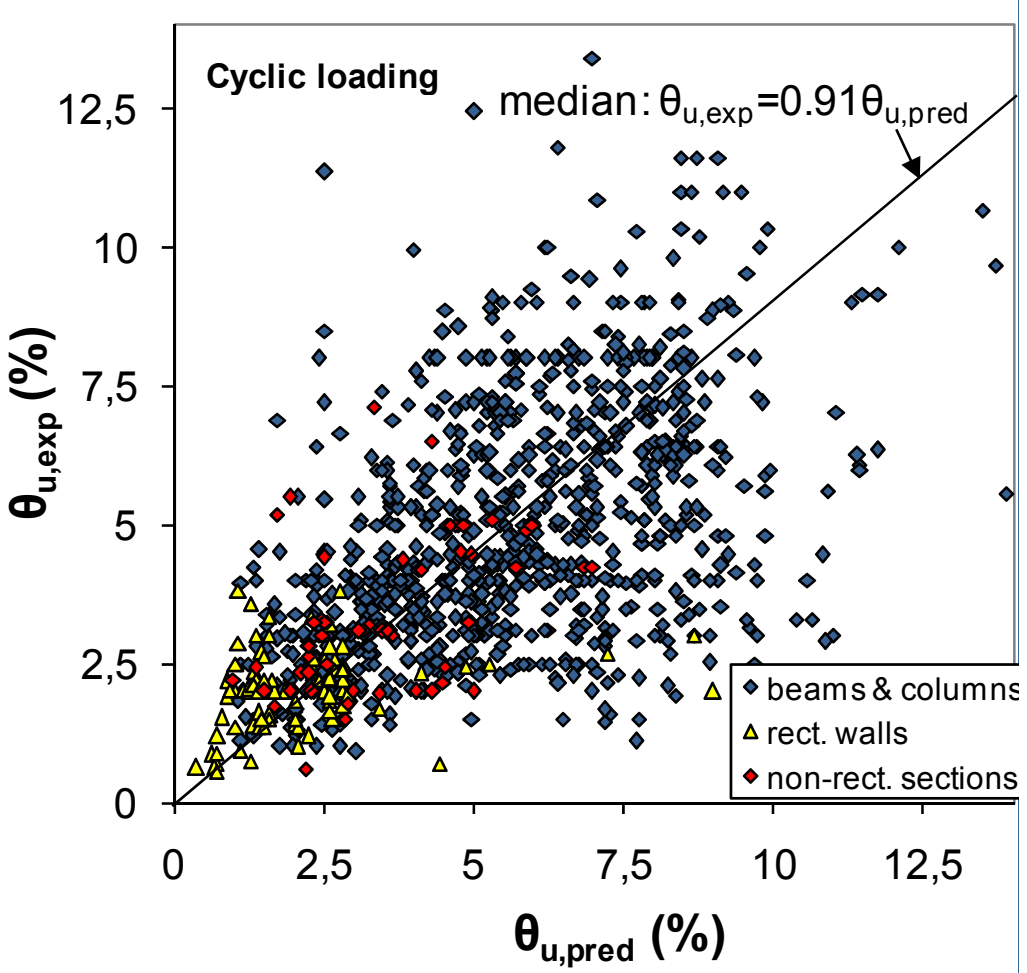
Option 1: confinement per EC2

median=0.88, CoV=52.3%

Option 2: new confinement per EC8-3

median=0.91, CoV=52.2%





Flexure-controlled ultimate chord rotation of rect. beams, columns, walls, non-rect. walls & circular columns from curvatures & plastic hinge length by Biskinis & Fardis 2010, 2013 - adopted in *fib*MC2010

Flexure-controlled ultimate chord rotation, accounting separately for slippage in yield-penetration length, from yielding till ultimate deformation:

$$\theta_u = \theta_y + a_{sl}\Delta\theta_{u,slip} + (\varphi_u - \varphi_y)L_{pl}\left(1 - \frac{L_{pl}}{2L_s}\right)$$

Confinement per *fib* MC2010:

$$f_{cc} = f_c\left[1 + 3.5\left(\frac{\alpha\rho_w f_{yw}}{f_c}\right)^{3/4}\right]$$

- **<u>Monotonic</u> loading - Rect. beams, columns, walls, non-rectangular walls:**

$$\varepsilon_{cu,c} = 0.0035 + \left(\frac{10}{h_o\,(mm)}\right)^2 + 0.57\frac{\alpha\rho_w f_{yw}}{f_{cc}},\quad \varepsilon_{su,mon} = \frac{7}{12}\varepsilon_{su,nominal}$$

$$L_{pl,mon} = h\left(1.1 + 0.04\min\left(9; \frac{L_s}{h}\right)\right)$$

- **<u>Cyclic</u> loading:**

$$\varepsilon_{cu,c} = 0.0035 + \left(\frac{10}{h_o\,(mm)}\right)^2 + 0.4\frac{\alpha\rho_w f_{yw}}{f_{cc}},\quad \varepsilon_{su,cy} = \frac{3}{8}\varepsilon_{su,nominal}$$

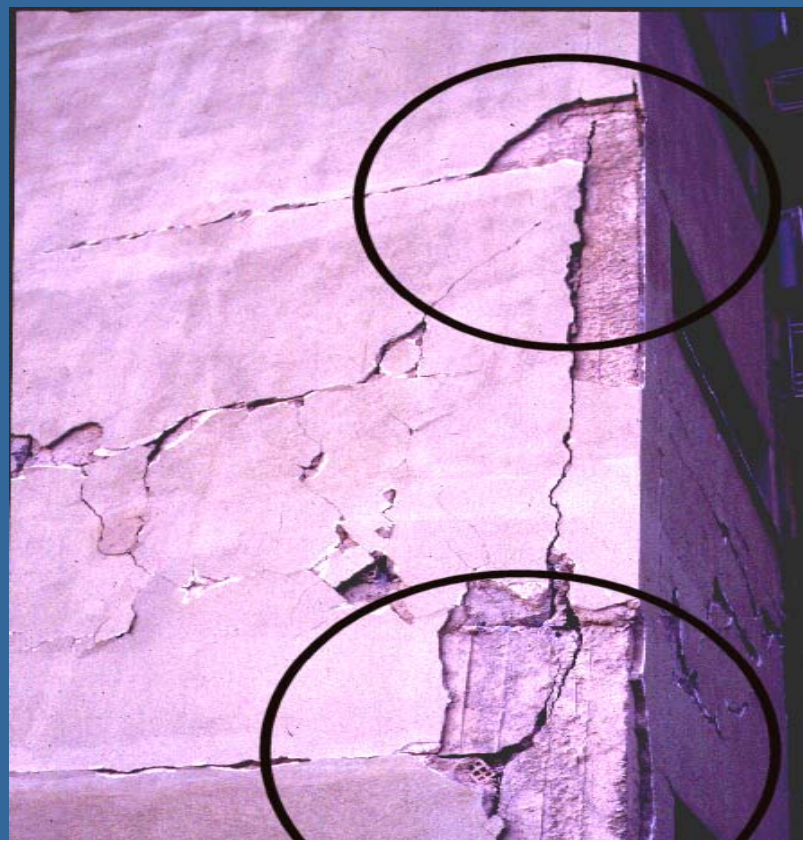
- **Rect. beams/columns/walls, non-rect. walls:** $L_{pl,cy} = 0.2h\left(1 + \frac{1}{3}\min\left(9; \frac{L_s}{h}\right)\right)$
- **Circular columns:** $L_{pl,cy,cir} = 0.6D\left(1 + \frac{1}{6}\min\left(9; \frac{L_s}{D}\right)\right)$

Post-yield fixed-end rotation of member end due to bar slippage from yield penetration length beyond member end, from yielding till ultimate flexural deformation per Biskinis & Fardis 2010, 2013 - adopted in *fib*MC2010 (not in EN 1998-3:2005)

- **Monotonic loading:** $\Delta\theta_{u,slip} = 9.5 d_{bL} \varphi_u \quad or \quad = 16 d_{bL} \left(\varphi_y + \varphi_u\right)/2$
- **Cyclic loading:** $\Delta\theta_{u,slip} = 5.5 d_{bL} \varphi_u \quad or \quad = 10 d_{bL} \left(\varphi_y + \varphi_u\right)/2$

Complete pull-out of beam bars, due to short anchorage in corner joint

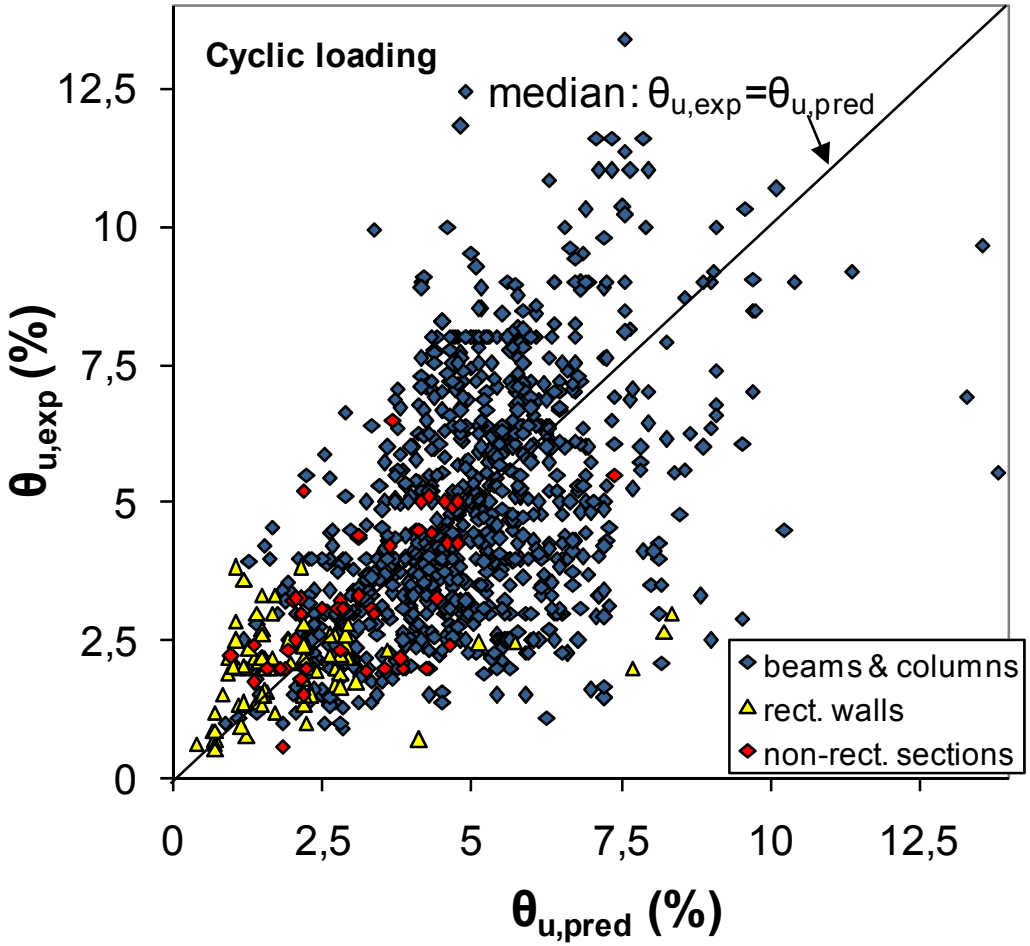
← →

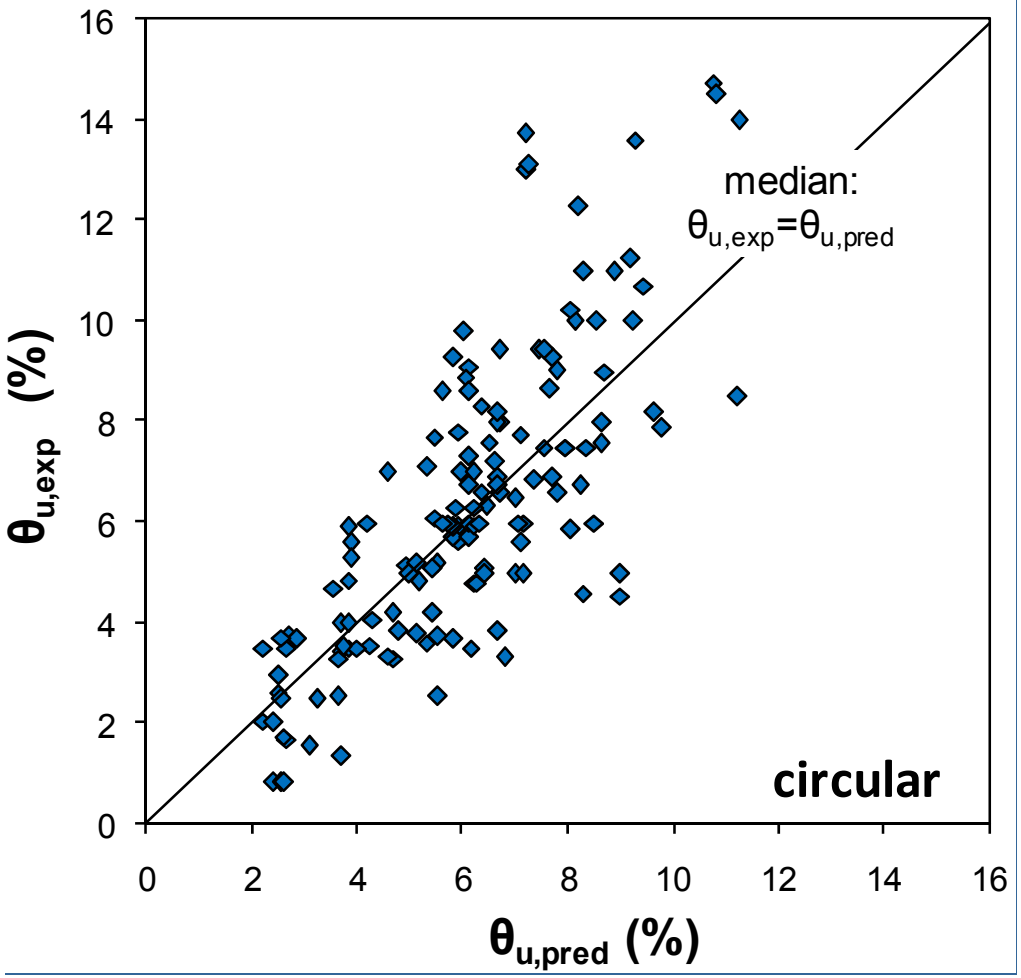


Test-model comparison – Cyclic ultimate chord rotation from curvatures & plastic hinge length per Biskinis & Fardis 2010, 2012

Rect. beams/columns/walls, non-rect. walls - no. tests: 1100 median=1.00, CoV=43.2%

Circular columns – no. tests: 143, median=1.00, CoV=30.3%





Empirical cyclic flexure-controlled ultimate chord rotation of rect. beams/columns/walls, non-rect. walls in EN1998-3:2005 & *fib*MC2010

$$\theta_{um} = 0.016 \cdot (0.3^{\nu}) \left[\frac{\max(0.01;\omega')}{\max(0.01;\omega)} f_c \right]^{0.225} \left(\frac{L_s}{h} \right)^{0.35} 25^{\left(\alpha\rho_{sx} \frac{f_{yw}}{f_c} \right)} (1.25^{100\,\rho_d})$$

or:

$$\theta_{um}^{pl} = \theta_{um} - \theta_y = 0.0145 \cdot (0.25^{\nu}) \left[\frac{\max(0.01;\omega')}{\max(0.01;\omega)} \right]^{0,3} f_c^{0.2} \left(\frac{L_s}{h} \right)^{0.35} 25^{\left(\alpha\rho_{sx} \frac{f_{yw}}{f_c} \right)} (1.275^{100\rho_d})$$

ω, ω': mechanical ratio of tension (including web) & compression steel;

ν: N/bhf_c (b: width of compression zone; N>0 for compression);

L_s/h : M/Vh: shear span ratio;

α : confinement effectiveness factor : $\alpha = \left(1 - \frac{s_h}{2b_c}\right)\left(1 - \frac{s_h}{2h_c}\right)\left(1 - \frac{\sum b_i^2}{6b_c h_c}\right)$

ρ_{sx}: $A_{sh}/b_w s_h$: transverse steel ratio // direction of loading;

ρ_d: ratio of diagonal reinforcement.

- Walls: 1st expression divided by 1.6; 2nd multiplied by 0.6
- Cold-worked brittle steel: 1st expression divided by 1.6; 2nd by 2.0

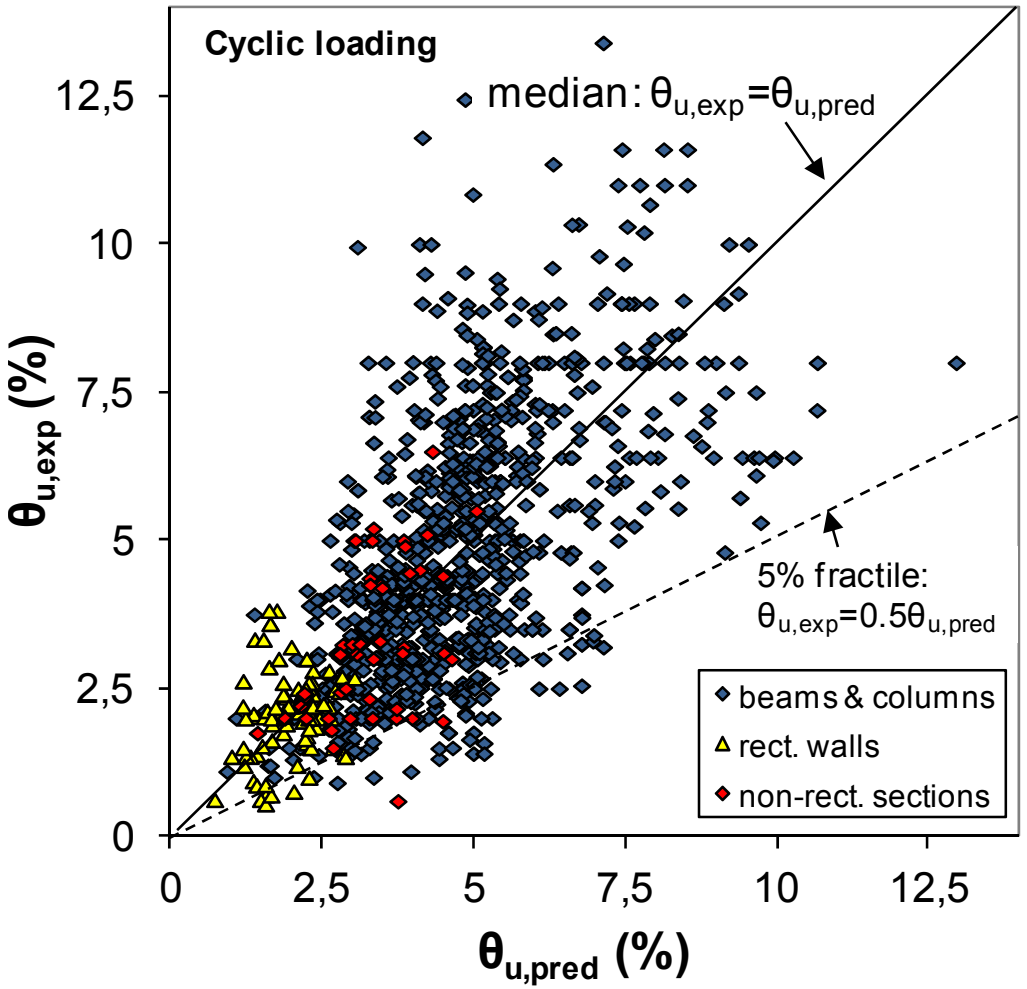
Non-seismically detailed members with continuous bars

- Plastic part, $\theta^{pl}_{um}=\theta_{um}-\theta_y$, of ultimate chord rotation: divided by 1.2.

Test-model comparison – Empirical cyclic ultimate chord rotation of members with seismic detailing per EN 1998-3:2005 – no. tests 1100

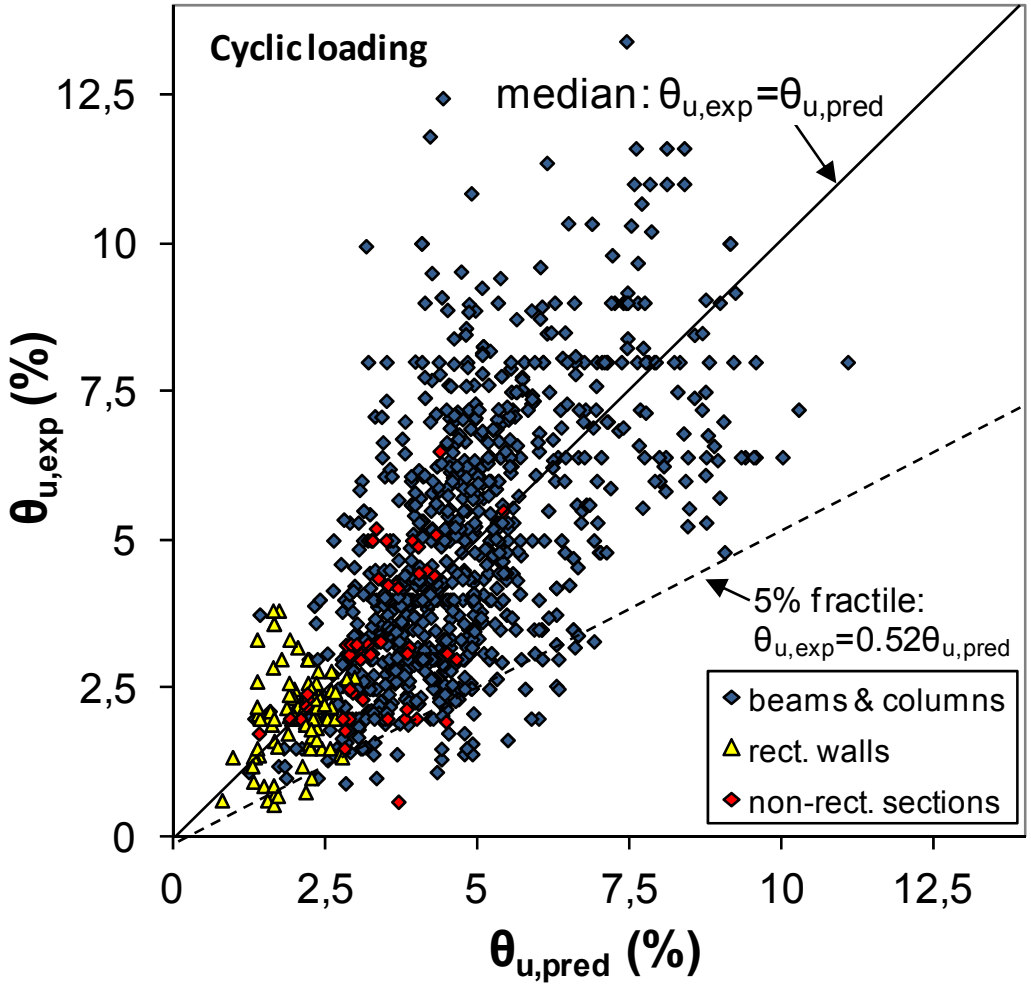
Model for total θ_{um}

median=1.00, CoV=37.8%



Model with $\theta_{um}=\theta_y+\theta_{pl}$

median=1.00, CoV=37.6%



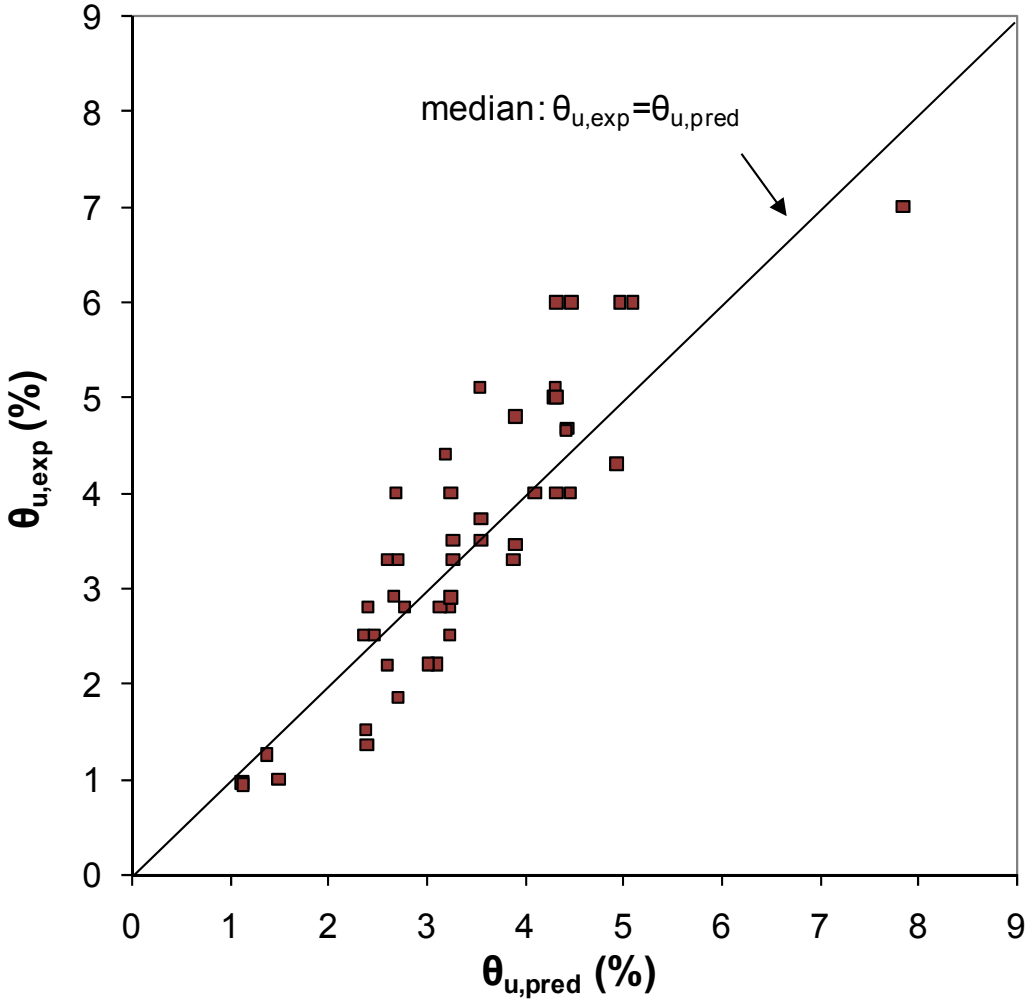
Test-model comparison – Empirical cyclic ultimate chord rotation of members without seismic detailing per EN 1998-3:2005 – no. tests 48

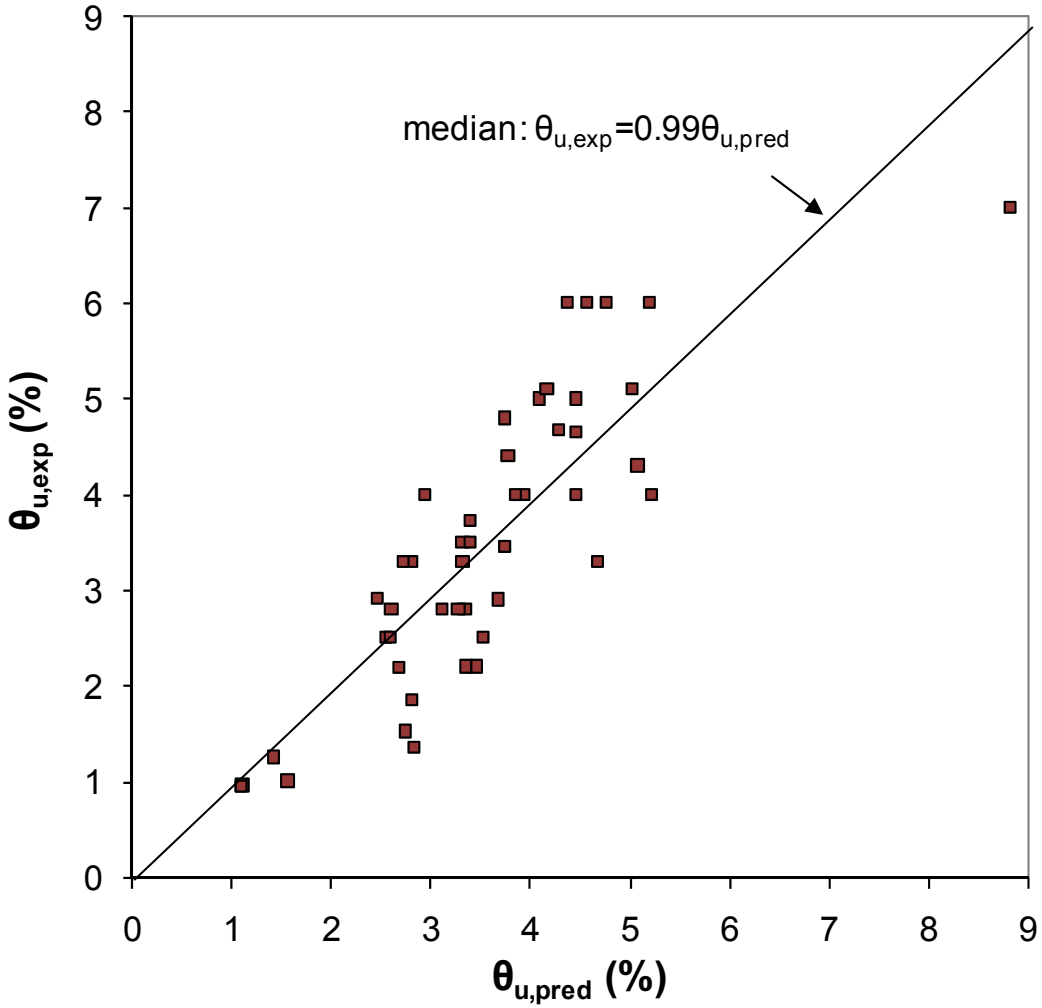
Model for total θ_{um}

median=1.00, CoV=30.6%

Model with $\theta_{um}=\theta_y+\theta_{pl}$

median=0.99, CoV=31.8%





General empirical ultimate chord rotation of rect. beams/columns/ walls & non-rect. walls per Biskinis & Fardis 2010, adopted in *fib* MC2010 (not in EN1998-3:2005)

$$\theta_u^{pl} = \alpha_{st}^{hbw}(1-0.525a_{cy})(1+0.6a_{sl})\left(1-0.052\max\left(1.5;\min\left(10;\frac{h}{b_w}\right)\right)\right)(0.2)^{\nu}\left(\frac{\max(0.01;\omega_2)}{\max(0.01;\omega_1)}\min\left(9;\frac{L_s}{h}\right)\right)^{\frac{1}{3}} f_c^{0.2}\, 25^{\left(\frac{\alpha\rho_w f_{yw}}{f_c}\right)} 1.225^{100\rho_d}$$

α_{st}^{hbw}:
0.022 for hot-rolled or Tempcore bars;
0.0095 for brittle cold-worked bars;

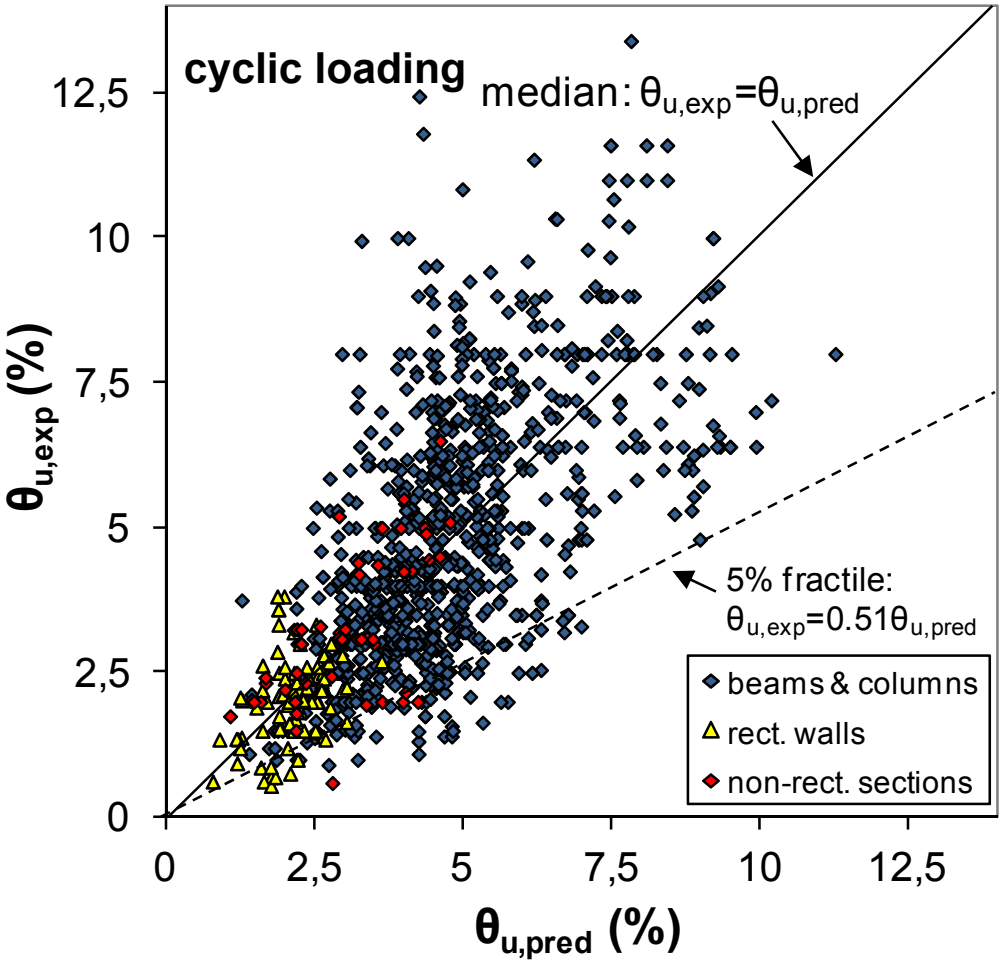
α_{cy}:
= 1 for cyclic loading,
= 0 for monotonic;

α_{sl}:
= 1 if slippage of long. bars from anchorage zone is possible,
= 0 otherwise;

b_w: width of (one) web

Non-seismically detailed members:

- $\theta^{pl}_{um}=\theta_{um}-\theta_y$ divided by 1.2.



no. tests: 1100, median=1.00, CoV=37.6%

Effect of lap-splicing the column bars in the plastic hinge region

Members with ribbed bars lap-spliced over length l_o inside the plastic hinge region per EN 1998-3:2005 or other options

- **EN 1998-3:2005:**

1. Both bars in pair of lapped compression bars count as compression steel.
2. For the yield properties (M_y, ϕ_y , θ_y), the stress f_s of tension bars is:

$$f_s = f_y(l_o/l_{oy,min}),\ \text{if } l_o < l_{oy,min}=(0.3f_y/\sqrt{f_c})d_b \qquad (f_y,\ f_c \text{ in MPa})$$

3. Ultimate chord rotation

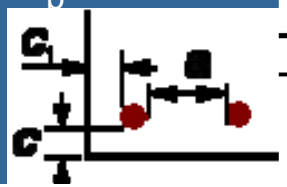
$$\theta_u=\theta_y+\theta^{pl}{}_u(l_o/l_{ou,min}),\ \text{if } l_o < l_{ou,min} = d_b f_y/[(1.05+14.5\alpha_{rs}\omega_{sx})\sqrt{f_c}],$$

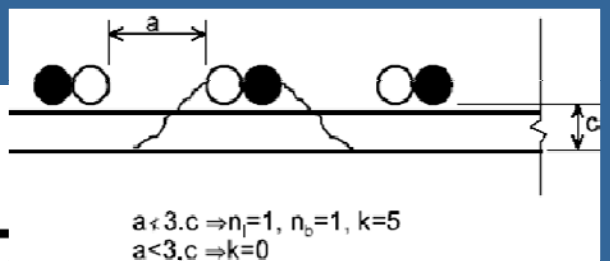
   - f_y, f_c in MPa, $\omega_{sx}=\rho_{sx}f_{yw}/f_c$: mech. transverse steel ratio // loading,
   - $\alpha_{rs}=(1-s_h/2b_o)(1-s_h/2b_o)n_{restr}/n_{tot}$ (n_{restr}/n_{tot}: restrained-to-total lap-spliced bars).

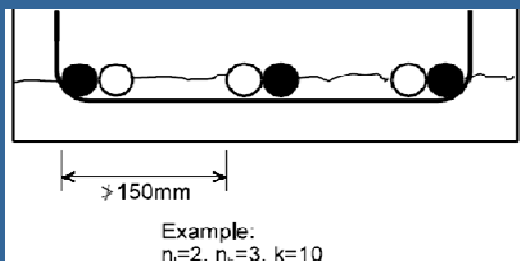
- **Or, Eligehausen & Lettow 2007 for *fib* MC2010**:

$$f_s = 51.2\left(\frac{l_b}{d_b}\right)^{0.55}\left(\frac{f_c}{20}\right)^{0.25}\left(\frac{20}{\max(d_b;\,20mm)}\right)^{0.2}\left[\left(\frac{c_d}{d_b}\right)^{1/3}\left(\frac{c_{\max}}{c_d}\right)^{0.1}+kK_{tr}\right] \le f_y,\ kK_{tr}=\frac{1}{n_b d_b}\left(\frac{k_s n_l A_{sh}}{s_h}\right)$$

- $c_d = \min[a/2;\ c_1;\ c] \ge d_b$, $c_d \le 3d_b$
- $c_{max} = \max[a/2;\ c_1;\ c] \le 5d_b$
- f_y, f_c in MPa

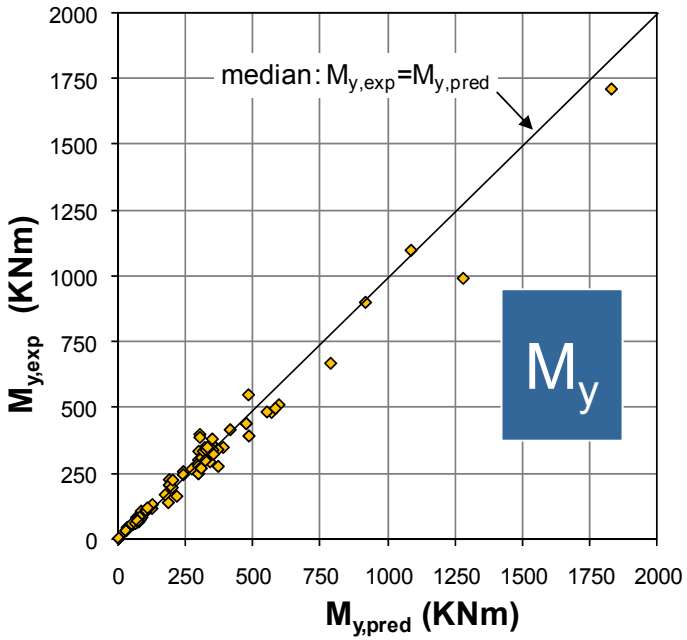




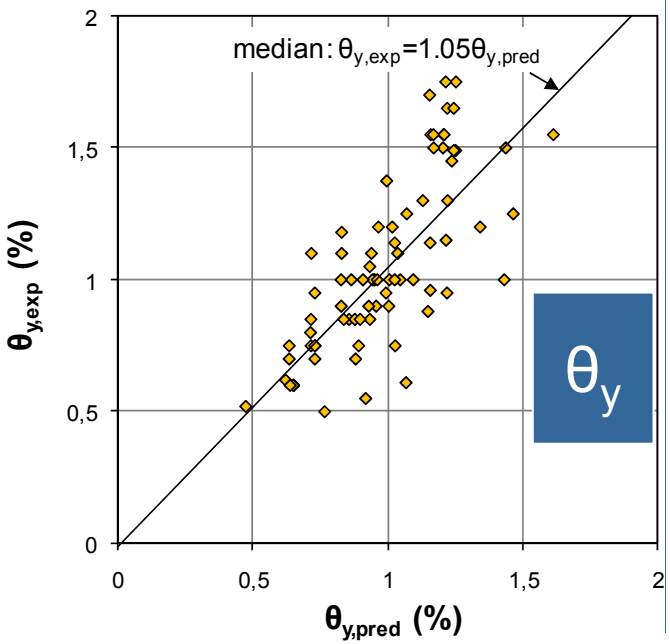


Test-model comparison: Yield moment & chord rotation, effective stiffness, ultimate cyclic chord rotation - lapped ribbed bars per EN 1998-3:2005

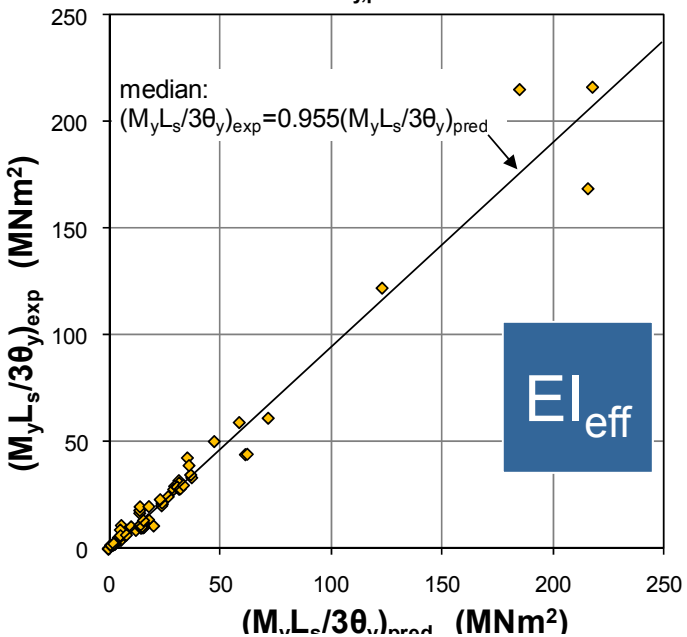
no.tests: 114
median=1.00
CoV=11.8%



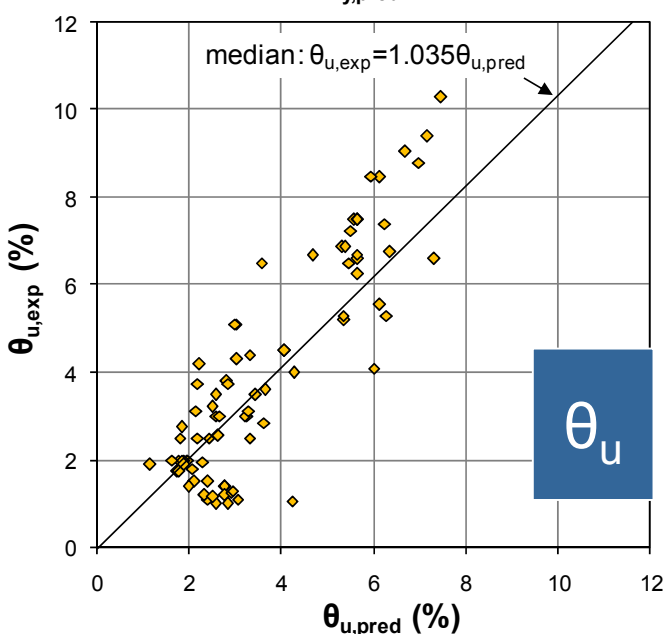
no.tests: 92
median=1.05
CoV=20%



no.tests: 92
median=0.955
CoV=24.8%

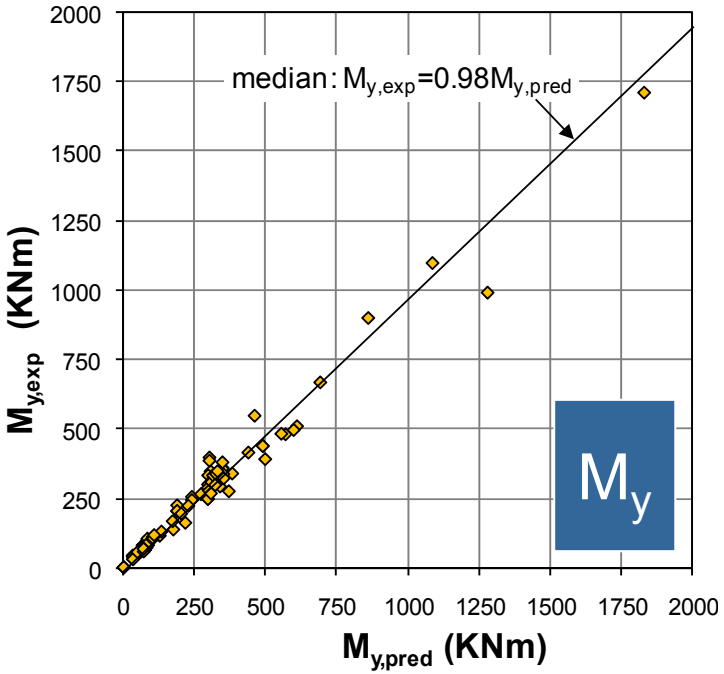


no.tests: 81
median=1.035
CoV=39.3%

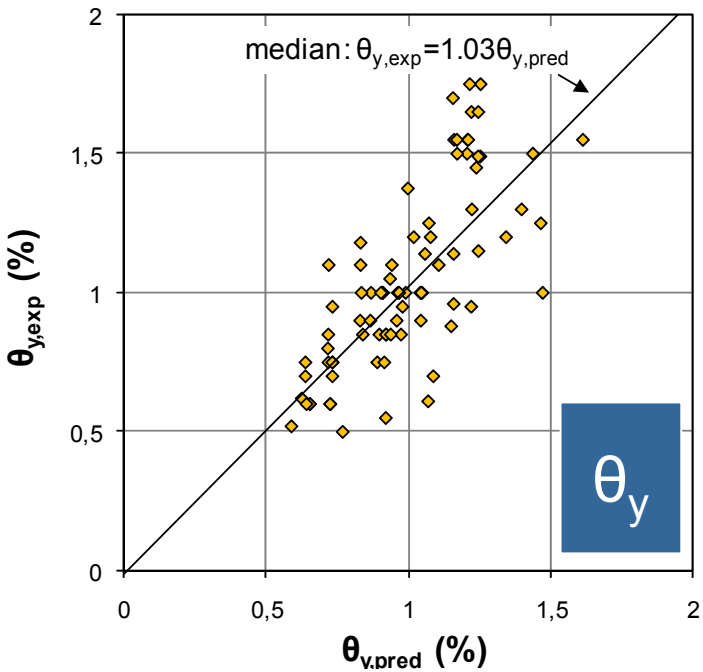


Test-model comparison: Yield moment & chord rotation, effective stiffness, - lapped bars, steel stress per Eligehausen & Letow 2007

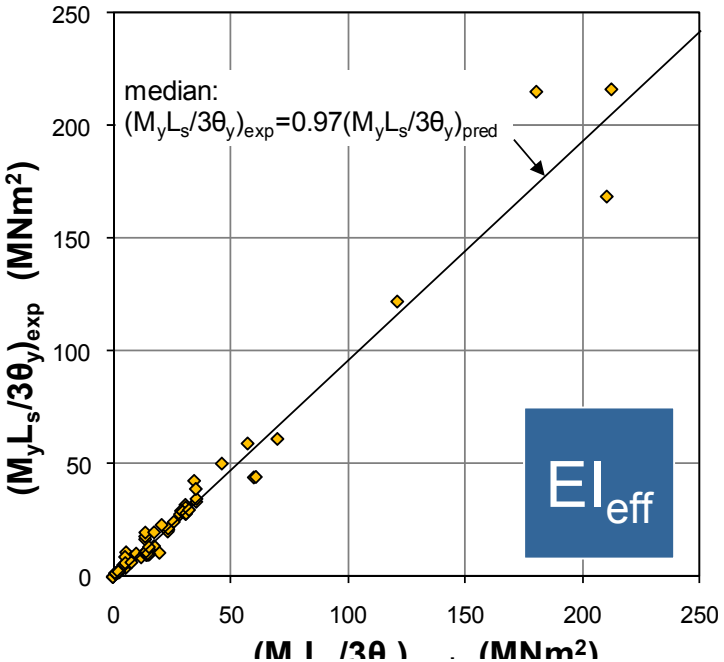
no.tests: 114
Median=0.98
CoV=12%



no.tests: 92
median=1.03
CoV=20.5%



no.tests: 92
median=0.97
CoV=24.6%



Members with smooth hooked bars, with or without lap splice (of length l_o) in the plastic hinge per EN 1998-3:2005

Provided that lapping $l_o \geq 15d_b$:

1. Yield properties M_y, ϕ_y, θ_y:
   - As in members with continuous ribbed bars.
2. Ultimate cyclic chord rotation:
   - For continuous bars:
     - $0.8\theta_{um}$ (: ~0.95 for smooth bars /1.2 for no seismic detailing) or
     - $\theta_{um}=\theta_y+0.75\theta^{pl}_{um}$ (: ~0.9 for smooth bars /1.2 for no seismic detailing)
   - For lapping **$l_o \geq 15d_b$** :
     - θ_{um} x $0.019(10+\min(40, l_o/d_b))$ or
     - $\theta_{um}=\theta_y+\theta^{pl}_{um}$ x $0.019 \min(40, l_o/d_b)$
   - L_s for θ^{pl}_{um} not reduced by l_o

(ultimate condition not necessarily controlled by region right above the lap)

Test-model-ratio for θ_{um}

	no laps	laps
Median	1.015	1.03
C.o.V.	33.3%	33.4%
no. tests	34	11

Cyclic shear resistance of RC members

Cyclic shear resistance per EN 1998-3:2005

- Shear resistance in pl. hinge after flex. yielding, as controlled by stirrups

(linear decay of V_c & V_w with cyclic plastic rotation ductility ratio $\mu_\theta^{pl}=(\theta-\theta_y)/\theta_y>0$

$$V_R=\frac{h-x}{2L_s}\min\left(N;\,0.55A_cf_c\right)+\left(1-0.05\min\left(5;\,\mu_\theta^{pl}\right)\right)\left[0.16\max(0.5;100\rho_{tot})\left(1-0.16\min\left(5;\frac{L_s}{h}\right)\right)\sqrt{f_c}A_c+V_w\right]$$

$V_w=\rho_w b_w z f_{yw}$, due to stirrups (b_w: web width, z: internal lever arm; ρ_w: shear reinf. ratio)

ρ_{tot}: total longitudinal reinforcement ratio

h: section depth

x : depth of compression zone at yielding

$A_c= b_w d$

- Shear resistance as controlled by web crushing (diagonal compression)
  - **Walls**, before flexural yielding ($\mu_\theta^{pl} = 0$) or after (cyclic $\mu_\theta^{pl} > 0$):

$$V_R=0.85\left(1-0.06\min\left(5,\,\mu_\Delta^{pl}\right)\right)\left(1+1.8\min\left(0.15,\frac{N}{A_cf_c}\right)\right)\left(1+0.25\max(1.75,100\rho_{tot})\right)\left(1-0.2\min\left(2,\frac{L_s}{h}\right)\right)\sqrt{f_c}b_wz$$

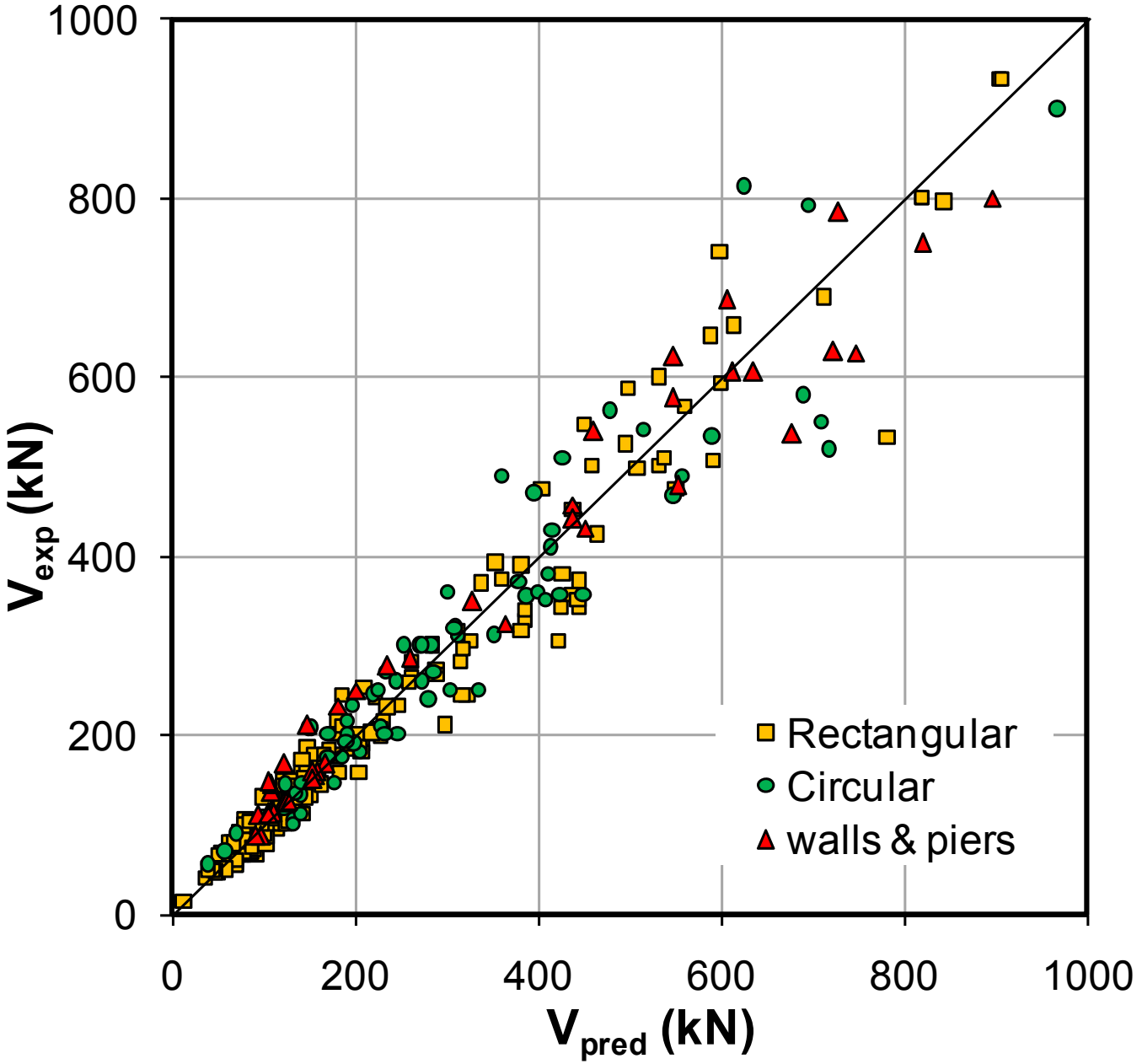
  - **Squat columns** ($L_s/h \le 2$) after flexural yielding (cyclic $\mu_\theta^{pl} > 0$):

$$V_R=\frac{4}{7}\left(1-0.02\min\left(5,\,\mu_\Delta^{pl}\right)\right)\left(1+1.35\frac{N}{A_cf_c}\right)\left(1+0.45\cdot100\rho_{tot}\right)\sqrt{\min\left(f_c,40\right)}b_wz\sin2\delta$$

δ: angle between axis and diagonal of column ($\tan\delta=0.5h/L_s$)

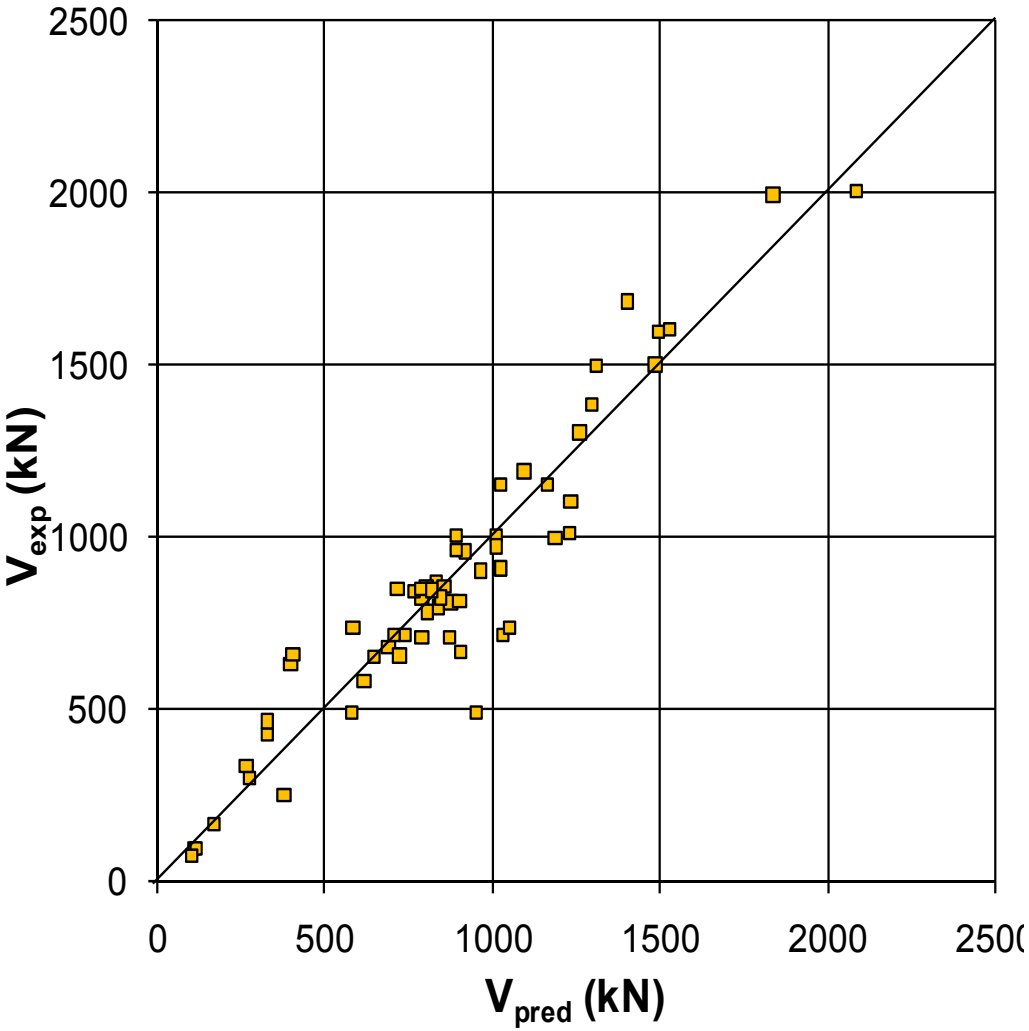
Test-model comparison: Cyclic shear resistance in plastic hinge (after flexural yielding) as controlled by stirrups, per EN 1998-3:2005

no. tests: 306
median=0.995
CoV=14.7%



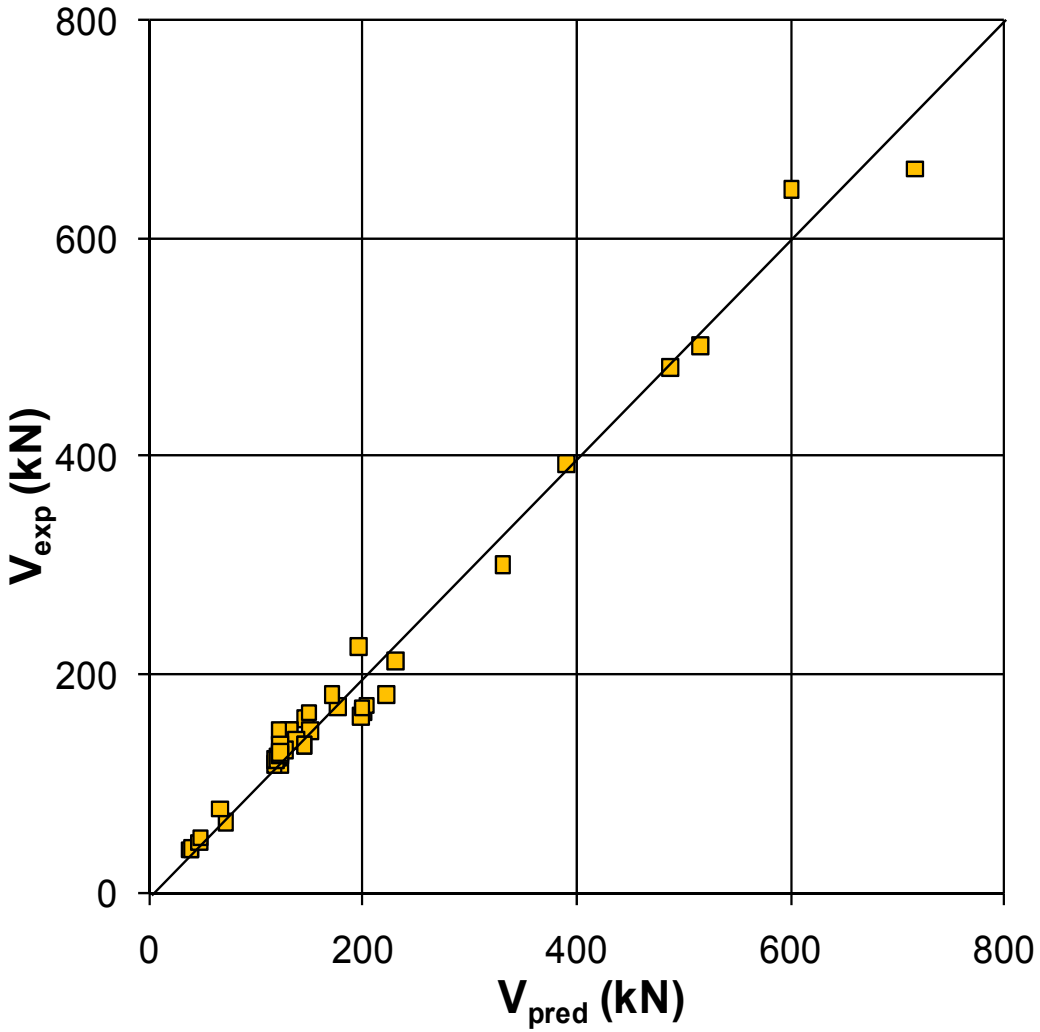
Test-model comparison: Cyclic shear resistance as controlled by web crushing (diagonal compression), per EN 1998-3:2005

Walls



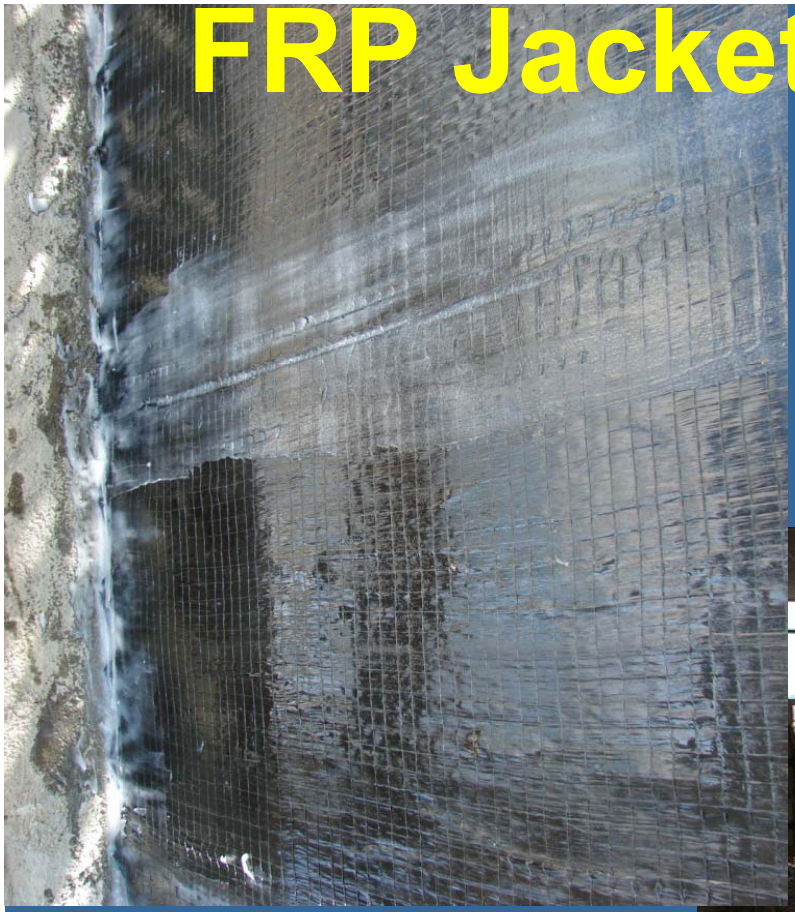
no. tests: 62, median=1.00, CoV=19.3%

Squat columns



no. tests: 40, median=1.00, CoV=9.6%

FRP Jackets per EN1998-3:2005



Experimental Database

4. Range and mean values of parameters in the tests of FRP-jacketed rectangular columns in the database

Parameter	219 columns with continuous bars (145 with CFRP, 24 with GFRP, 27 with AFRP and 23 with other composite)		45 columns with lap-spliced bars (42 with CFRP, 3 with GFRP)	
	min-max	mean	min-max	mean
effective depth, d (mm)	170-720	296	180-720	393
shear-span-to-depth ratio, L_s/h	1-7.4	3.55	2-6.6	4.5
concrete strength, f_c (MPa)	10.6-90	31.8	11.7-55	31
vertical bar yield stress, f_y (MPa)	295-816	431	331-617	492
stirrup yield stress, f_{yw} (MPa)	200-750	388	280-535	442
axial-load-ratio, N/A_cf_c	0-0.85	0.255	0-0.4	0.143
transverse steel ratio, ρ_w (%)	0-1.18	0.24	0-0.445	0.212
total vertical steel ratio, ρ_{tot} (%)	0.815-7.6	2.08	0.815-3.9	1.88
geometric ratio of FRP, ρ_f (%)	0.01-5.31	0.605	0.13-7.5	1.04
nominal FRP strength (MPa)	113-4830	2755	532-4430	2621
elastic modulus of FRP, E_f (GPa)	5.8-390	166	17.8-390	184
lapping-to-bar-diameter ratio, l_o/d_b	-	-	15-45	30.4

FRP-wrapping of plastic hinges in rectangular members with continuous bars per EN 1998-3:2005

- M_R, M_y: Enhanced by FRP jacket (by 9% w.r.to calculated w/o confinement)
  - EN 1998-3:2005: increase neglected.
- Effective (elastic) stiffness EI_{eff}: unaffected by FRP; pre-damage: 35% drop
- EN1998-3:2005: Flexure-controlled ultimate chord rotation, θ_u:
  - Confinement by FRP increases that due to the stirrups by $\alpha_f \rho_f f_{f,e}/f_c$, where:
    - $\rho_f = 2t_f/b_w$: FRP ratio // direction of loading;
    - $f_{f,e}$: FRP effective strength:

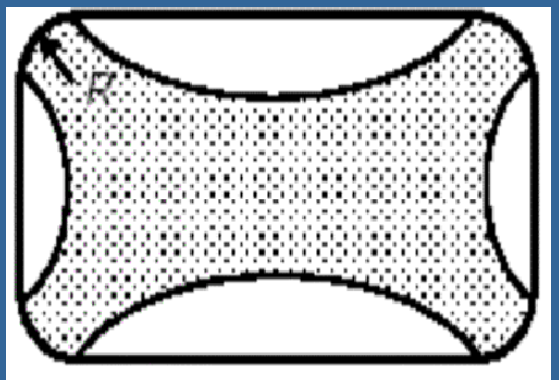
$$f_{f,e} = \min\left(f_{u,f}, \varepsilon_{u,f} E_f\right) \min\left(0.5; 1 - 0.7 \frac{\min\left(f_{u,f}, \varepsilon_{u,f} E_f\right)\rho_f}{f_c}\right)$$

$f_{u,f}$, E_f : FRP tensile strength & Modulus;
$\varepsilon_{u,f}$: FRP limit strain. CFRP/AFRP: $\varepsilon_{u,f}$=1.5%; GFRP: $\varepsilon_{u,f}$=2%

- FRP-confinement effectiveness:

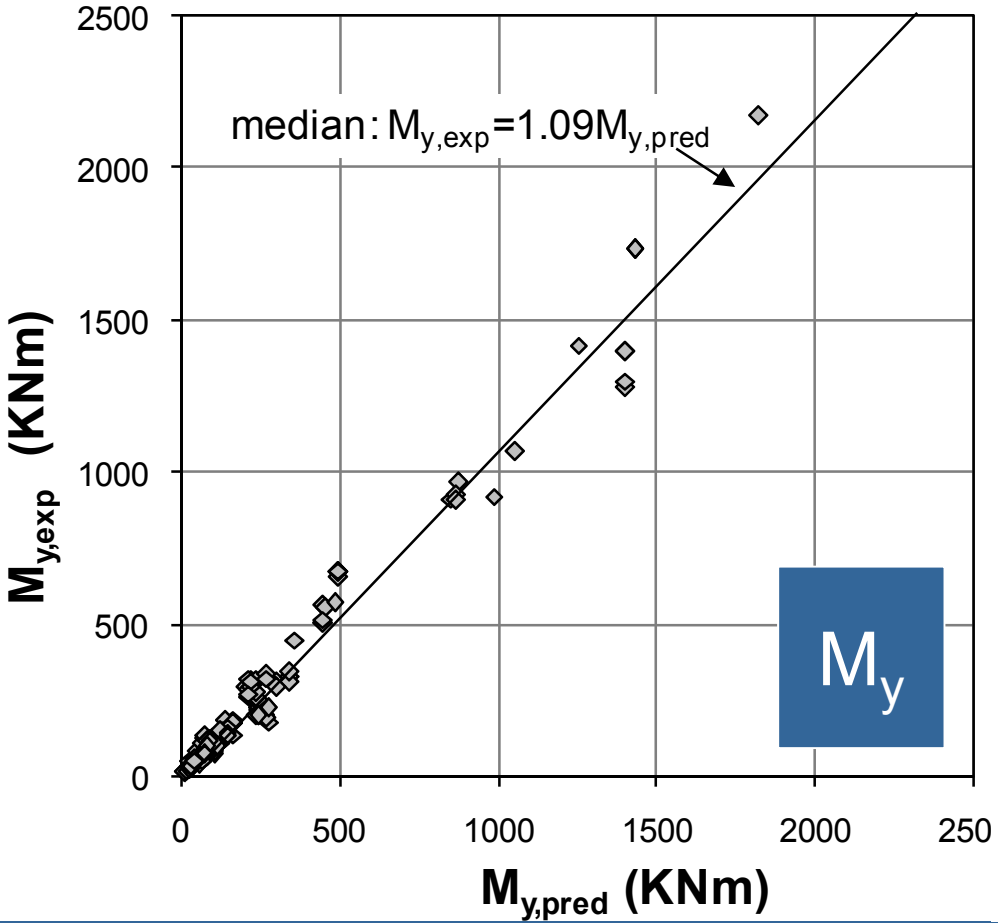
$$\alpha_f = 1 - \frac{(h-2R)^2 + (b-2R)^2}{3bh}$$

b, h: sides of section;
R: radius at section corner

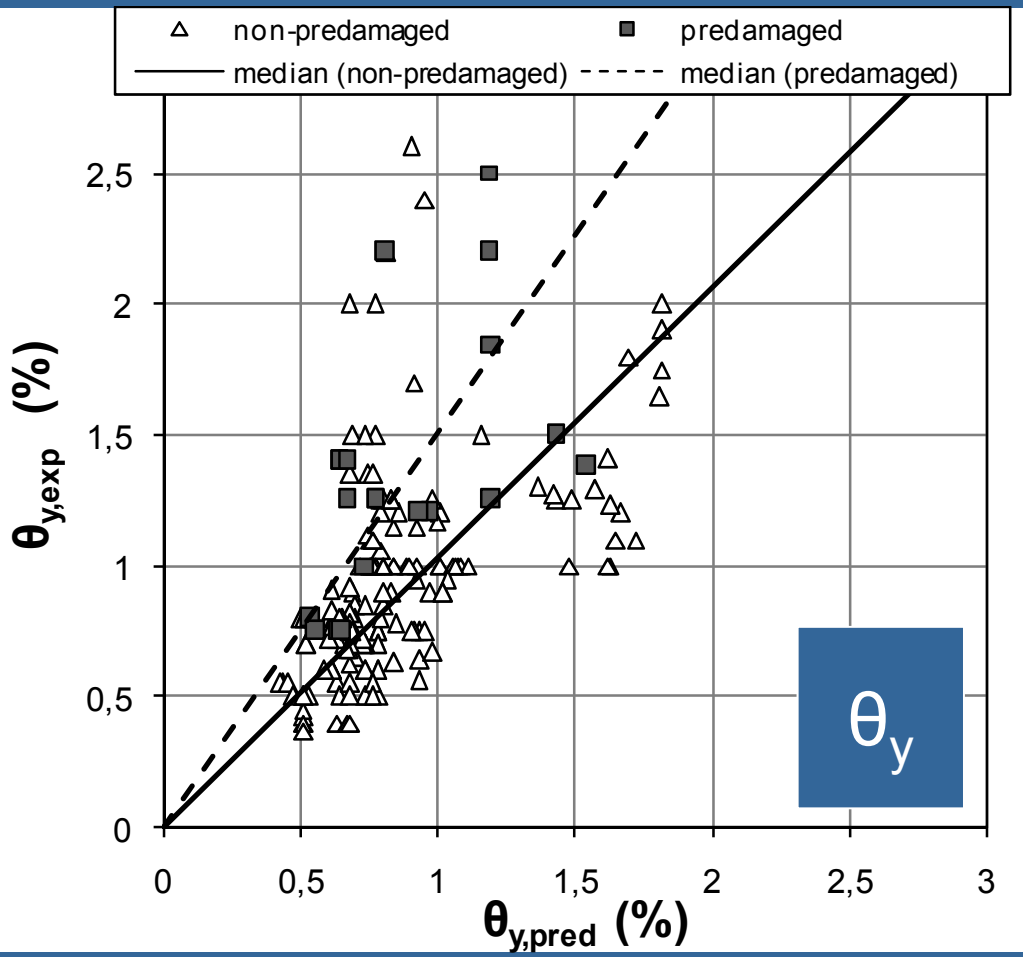


Test-model comparison - yield properties for FRP-wrapping of rectangular columns with continuous bars per EN 1998-3:2005

no.tests: 203 (pre-damaged or not)
median=1.09, CoV=19.4%



no.tests: 159 (no pre-damage)
median=1.03, CoV=37.3%



EI_{eff} (no pre-damage), no.tests: 159, median=1.02, CoV=30%

Yield properties for FRP-wrapping of the plastic hinge and continuous bars – Biskinis & Fardis 2007, 2013

- **Yield moment, M_y:**
  - Strength of FRP-confined concrete f_{cc}, instead of f_c, in section-analysis for φ_y, M_y, including the calculation of the concrete Elastic Modulus, E_c;
  - E_c may be estimated per ***fib***MC2010, using f_{cc} instead of f_c: $E_c=10000(f_{cc}(\text{MPa}))^{1/3}$
  - f_{cc} from (widely used) Lam & Teng 2003:

$$\frac{f_{cc}}{f_c}=1+3.3\left(\frac{b_x}{b_y}\right)^2\frac{a_f\rho_f f_{u,f}}{f_c}$$

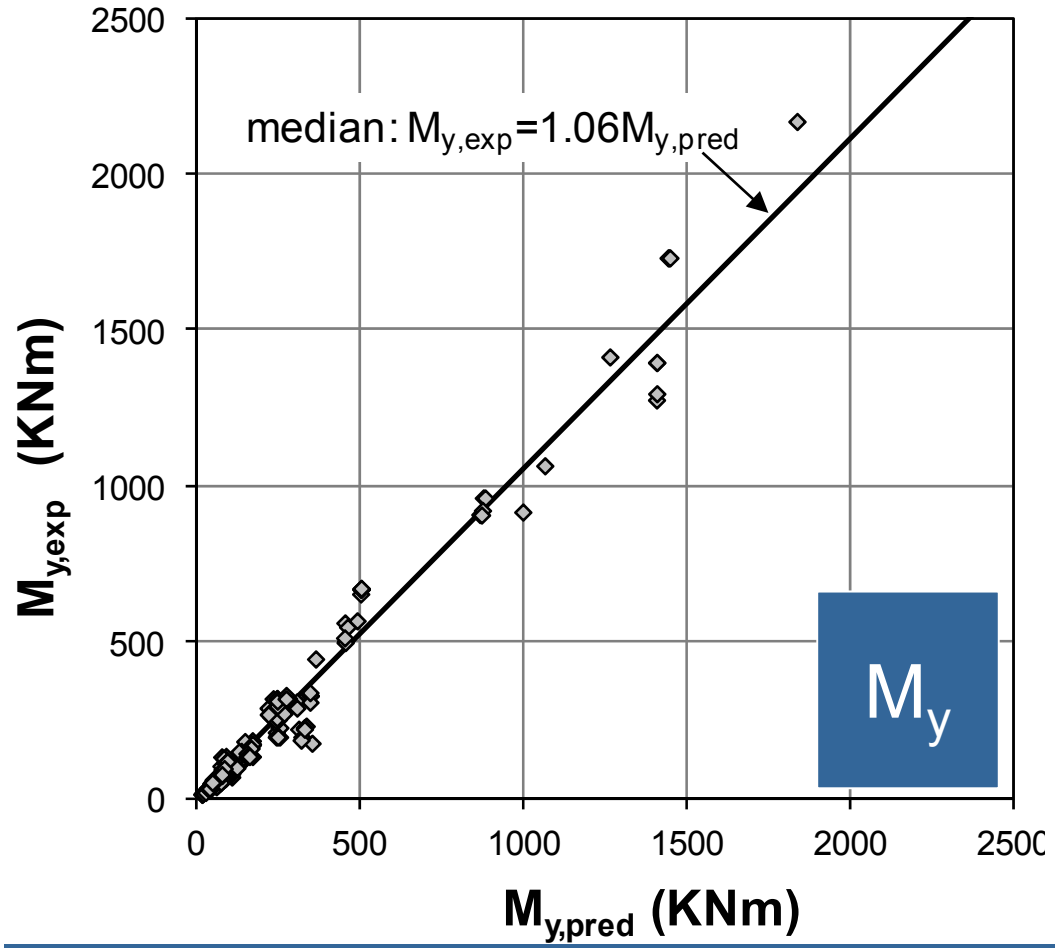
    $f_{u,f}$: effective strength of FRP: $f_{u,f}=E_f(k_{eff}\varepsilon_{u,f})$
    E_f: Elastic Modulus of FRP,
    $\varepsilon_{u,f}$: FRP failure strain,
    k_{eff}: FRP effectiveness factor, equal to 0.6 per Lam & Teng

  (~same results using other FRP-confinement models: Teng et al 2009, Samaan et al 1998, Bisby et al 2005, Ilki et al 2008, Wang et al 2012)

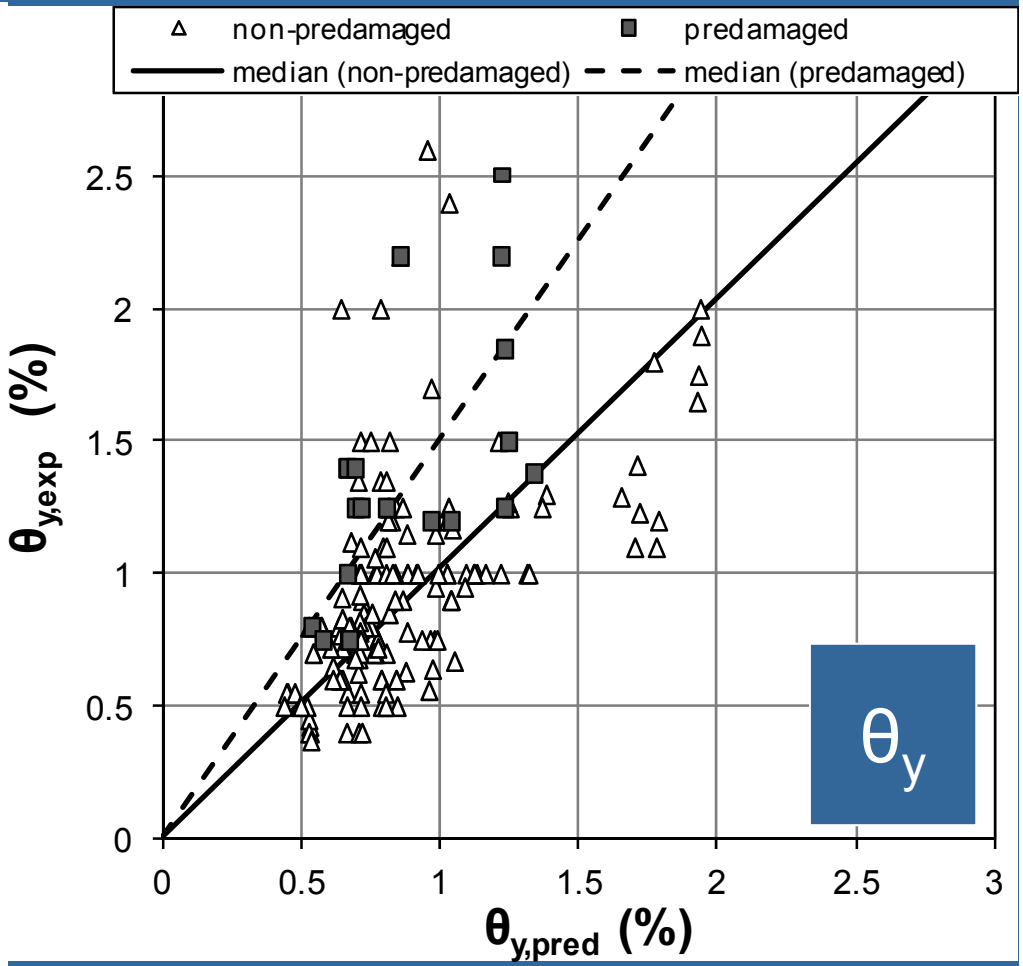
- **Chord rotation at yielding, θ_y, effective stiffness, EI_{eff} :**
  - Yield curvature φ_y calculated with f_{cc} (as above) & multiplied times correction factor 1.06
  - $EI_{eff} = M_y L_s/3\theta_y$

Test-model comparison - yield properties for FRP-wrapping of rect. columns with continuous bars per Biskinis & Fardis 2007, 2013

no.tests: 203 (pre-damaged or not)
median=1.06, CoV=19.3%



no.tests: 159 (no pre-damage)
median=1.01, CoV=37.3%



EI_{eff} (no pre-damage): no.tests: 159, median=0.99, CoV=30.4%

Extension of empirical ultimate plastic chord rotation of rect. columns with continuous bars & FRP-wrapping - Biskinis & Fardis 2007, 2013

- **Pre-damaged or not:**

$$\theta_u^{pl} = 0.0185 \cdot (1 - 0.52 a_{cy}) \left(1 + \frac{a_{sl}}{1.6}\right) (0.25)^{v} \left(\frac{\max(0.01, \omega')}{\max(0.01, \omega)}\right)^{0.3} f_c^{0.2} \left(\frac{L_s}{h}\right)^{0.35} 25^{\left[\frac{\alpha\rho_w f_{yw}}{f_c} + \left(\frac{\alpha\rho f_u}{f_c}\right)_{f,eff}\right]} 1.275^{100\rho_d}$$

with:

$$\left(\frac{a\rho f_u}{f_c}\right)_{f,eff} = a_f c_f \min\left[0.4; \frac{\rho_f f_{u,f}}{f_c}\right] \left(1 - 0.5 \min\left[0.4; \frac{\rho_f f_{u,f}}{f_c}\right]\right)$$

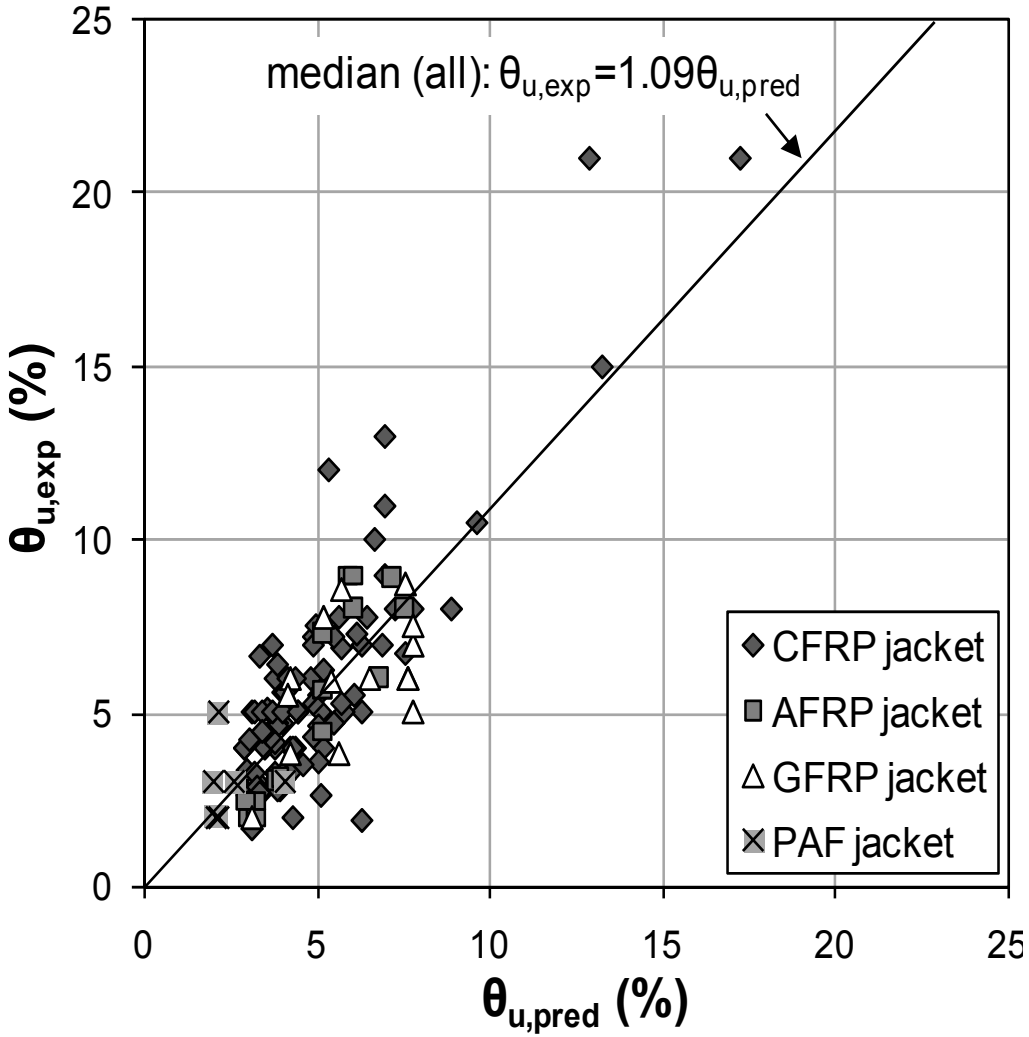
c_f=1.8 for CFRP or polyacetal fiber (PAF) sheets,

c_f=0.8 for GFRP or AFRP.

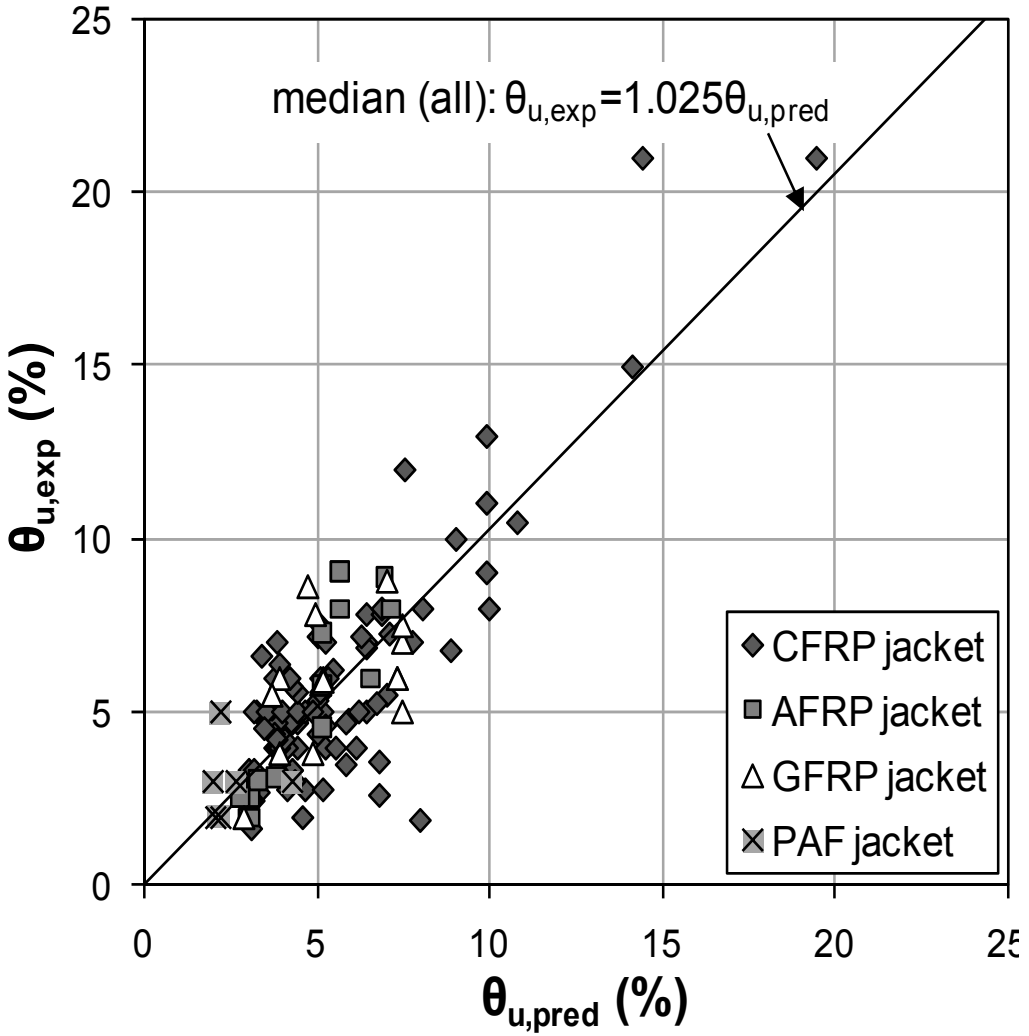
$f_{u,f}=E_f(k_{eff}\varepsilon_{u,f})$: effective FRP strength

Test-model comparison – ultimate cyclic chord rotation for FRP-wrapping of rect. columns with continuous bars: no. tests 128 (pre-damaged or not)
EN 1998-3:2005 v Biskinis & Fardis 2007, 2013

EN 1998-3:2005
median=1.09, CoV=30.6%



Biskinis & Fardis 2013
median=1.025, CoV=30.4%



Alternative ultimate cyclic chord rotation for continuous bars & FRP-wrapping – per GCSI (KANEPE)

$$\frac{f_{cc}}{f_c} = 1.125 + 1.25\alpha\omega_{wd}$$

ω_{wd}: FRP volumetric ratio
α: FRP-confinement effectiveness factor

CFRP (adopted here also for AFRP and PAF): $\varepsilon_{cu} = 0.0035\left(f_{cc}/f_c\right)^2$

GFRP: $\varepsilon_{cu} = 0.007\left(f_{cc}/f_c\right)^2$

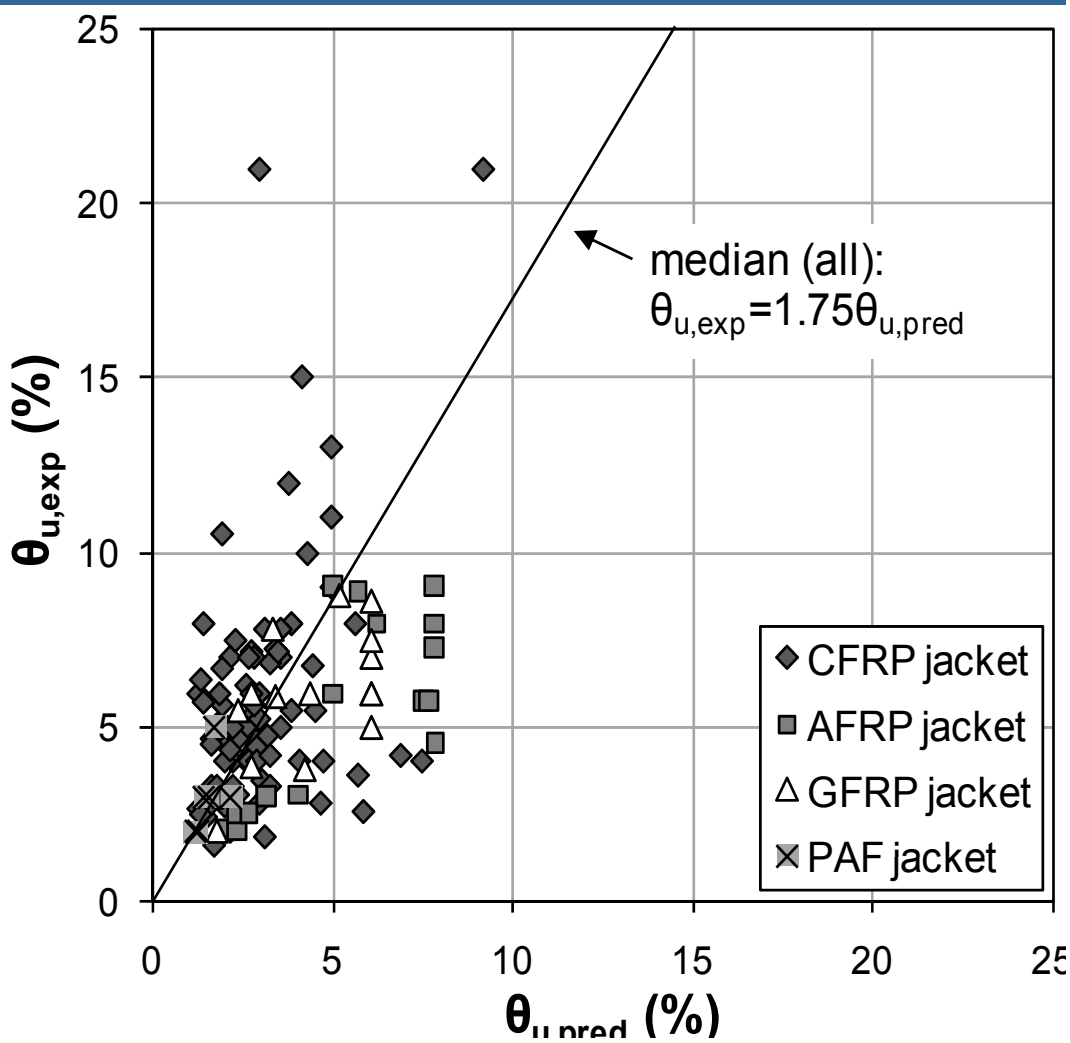
FRP effective strength: $f_{f,e} = f_{u,f}\psi$

ψ: reduction factor for number of layers (k);
ψ=1 for k<4

$\psi = k^{-1/4}$ for k≥4

$$\mu_\theta = \frac{\theta_u}{\theta_y} \cong \mu_\delta = \left(\mu_\varphi + 2\right)/3$$

Test-to-predicted ultimate cyclic chord rotation, θ_u – no.tests: 128, median=1.75, CoV=54.5%



FRP-wrapped rectangular members with ribbed bars lap-spliced over length l_o in the plastic hinge: EN 1998-3:2005 & other options

- **EN 1998-3:2005:**

1. Both bars in pair of lapped compression bars count as compression steel.
2. For the yield properties (M_y, ϕ_y , θ_y), the stress f_s of tension bars is:

   $f_s=f_y(l_o/l_{oy,min})$, if $l_o< l_{oy,min}=(0.2f_y/\sqrt{f_c})d_b$ (f_y, f_c in MPa), $l_{oy,min}$: one-third shorter than without FRP.
3. Ultimate chord rotation

   $$\theta_u=\theta_y+\theta^{pl}{}_u(l_o/l_{ou,min}),\ \text{if } l_o < l_{ou,min} = d_b f_y/[(1.05+14.5\,\alpha_l\rho_f f_{f,e})\sqrt{f_c}]$$

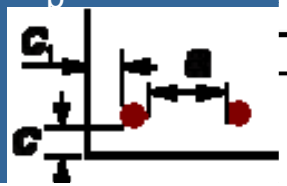
   – f_c: MPa, $\rho_f=2t_f/b_w$: FRP ratio // loading, $f_{f,e}$: effect. FRP strength (MPa),

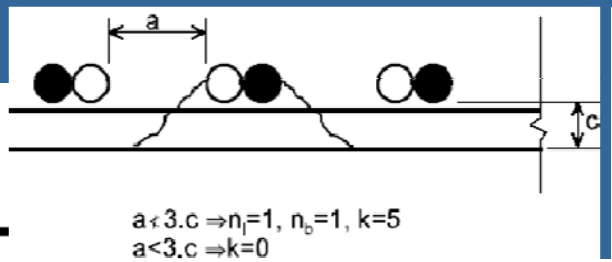
   – $\alpha_l = \alpha_f(4/n_{tot})$ (n_{tot}: total lap-spliced bars; only the 4 corner bars restrained).

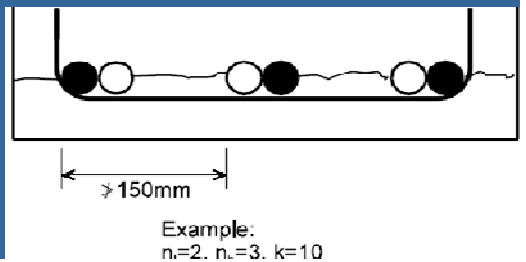
- **Or, Eligehausen & Lettow 2007 for *fib*MC2010**:

$$f_s = 51.2\left(\frac{l_b}{d_b}\right)^{0.55}\left(\frac{f_c}{20}\right)^{0.25}\left(\frac{20}{\max(d_b;20mm)}\right)^{0.2}\left[\left(\frac{c_d}{d_b}\right)^{1/3}\left(\frac{c_{\max}}{c_d}\right)^{0.1}+kK_{tr}\right]\le f_y,\quad kK_{tr}=\frac{1}{n_b d_b}\left(\frac{k_s n_l A_{sh}}{s_h}+\frac{k_f n_f t_f E_f}{E_s}\right)$$

- $c_d = \min[a/2;\ c_1;\ c] \ge d_b$, $c_d \le 3d_b$
- $c_{max} = \max[\alpha/2;\ c_1;\ c] \le 5d_b$
- f_y, f_c in MPa



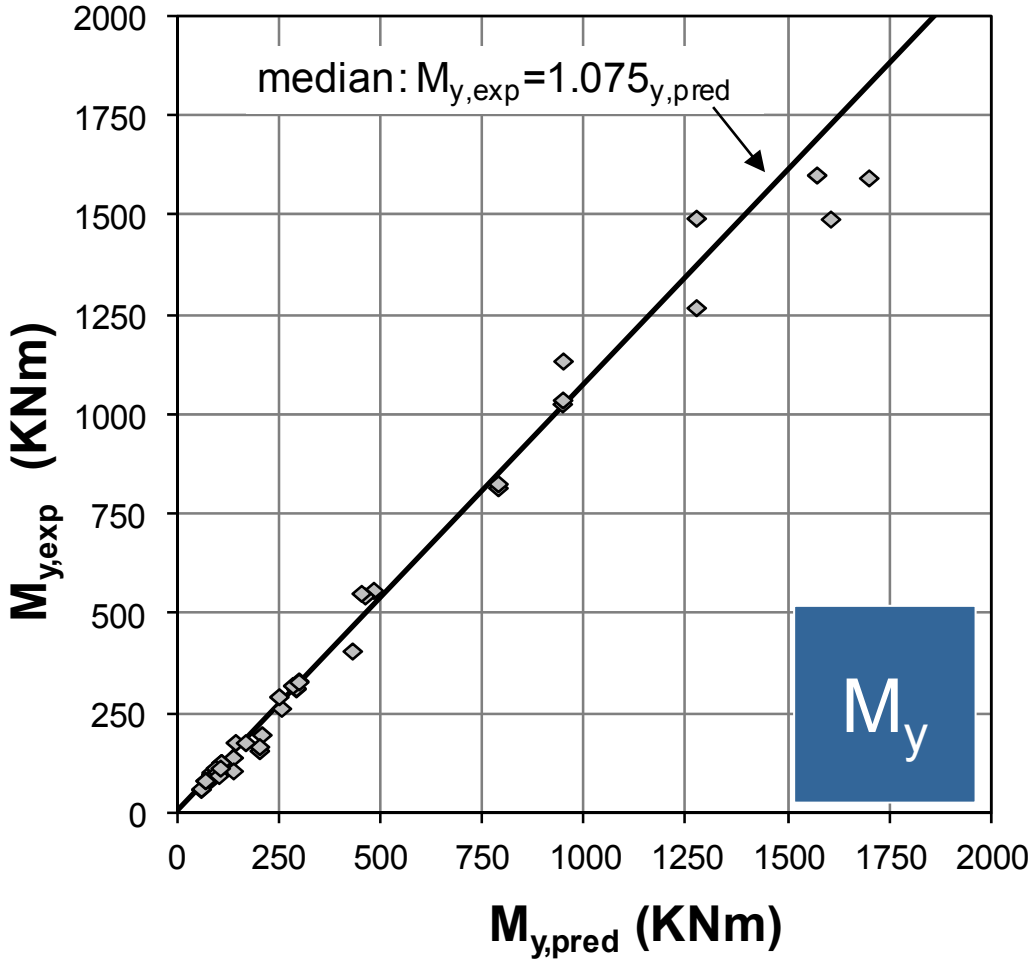


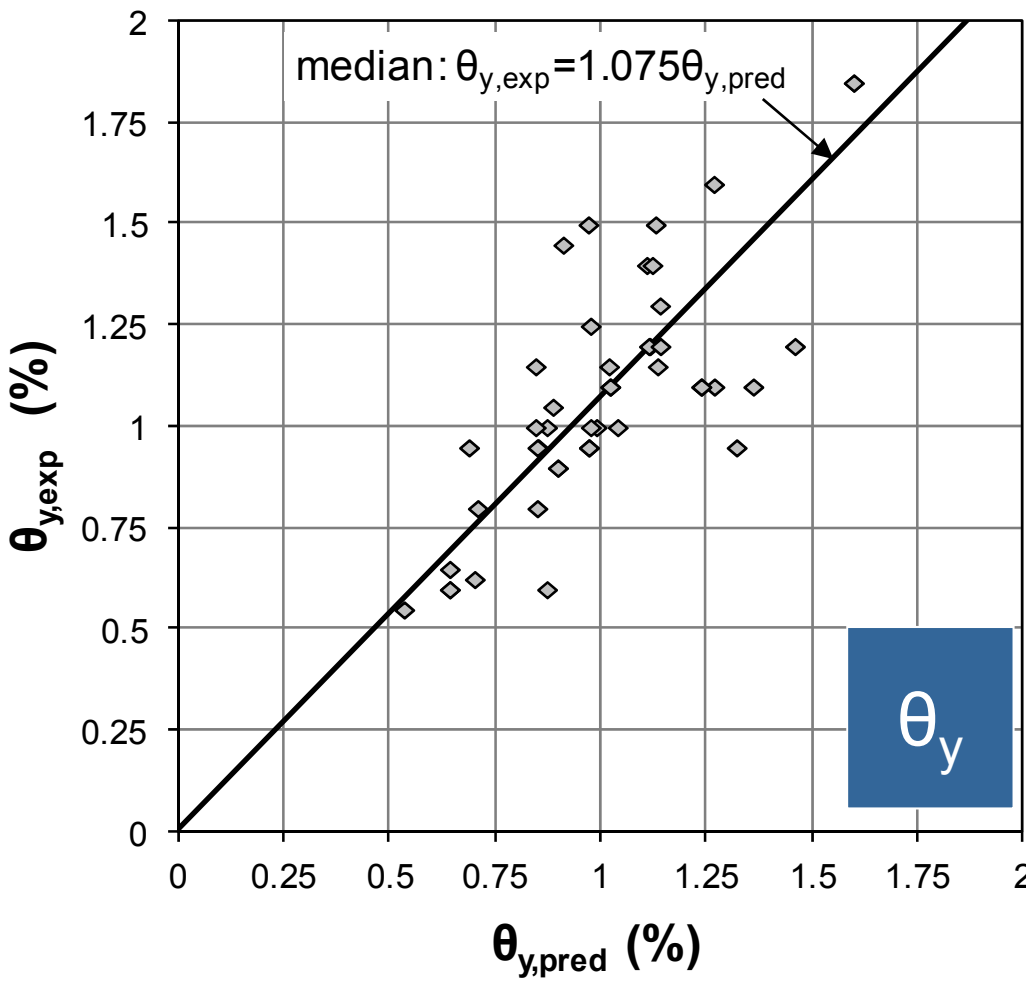


Test-model comparison - Yield properties for FRP-wrapping of rect. columns with bars lap-spliced over length l_o per EN1998-3:2005, no.tests 45

M_y: median=1.075, CoV=10.5%

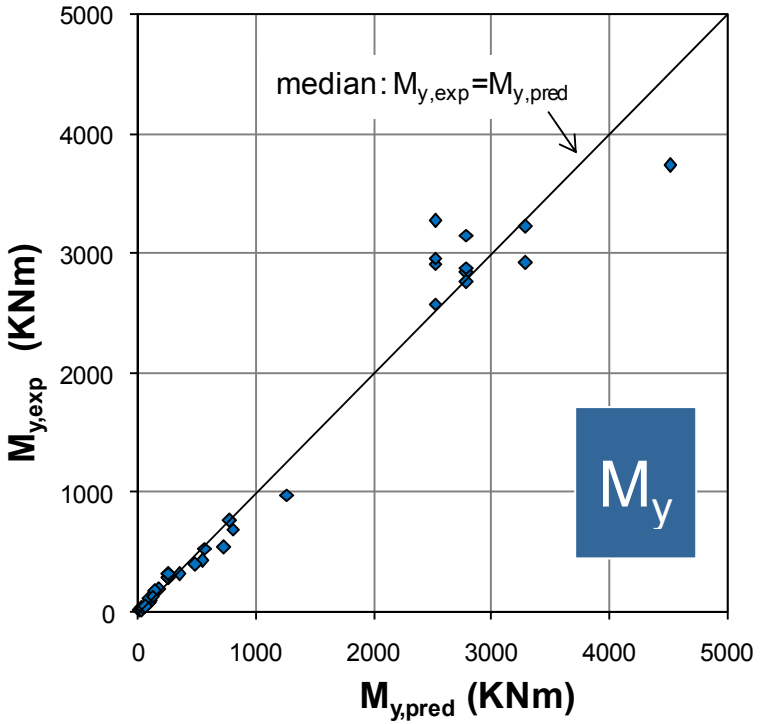
θ_y: median=1.075, CoV=17.7%



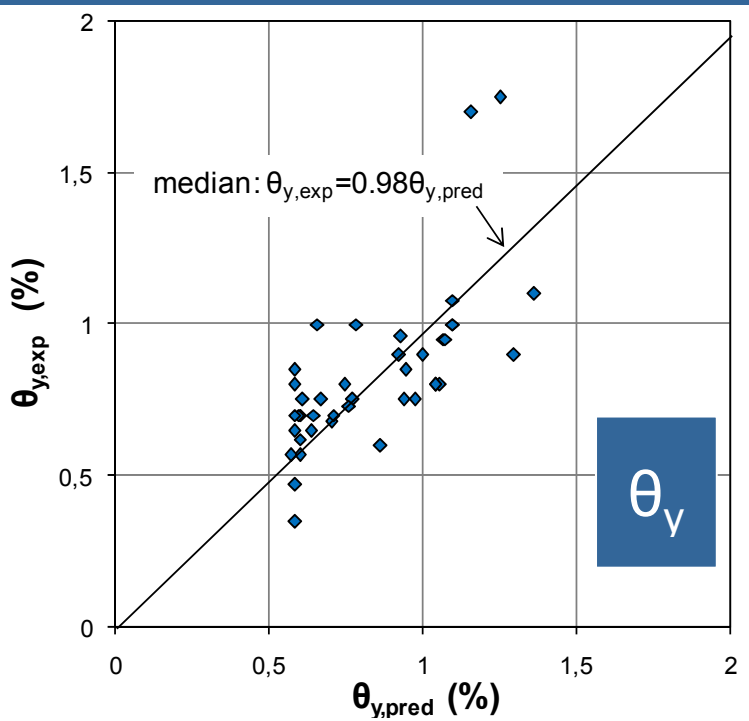


Test-model comparison - yield properties of FRP-wrapped circular columns with lap-spliced bars per EN1998-3:2005 (Biskinis & Fardis 2007, 2013)

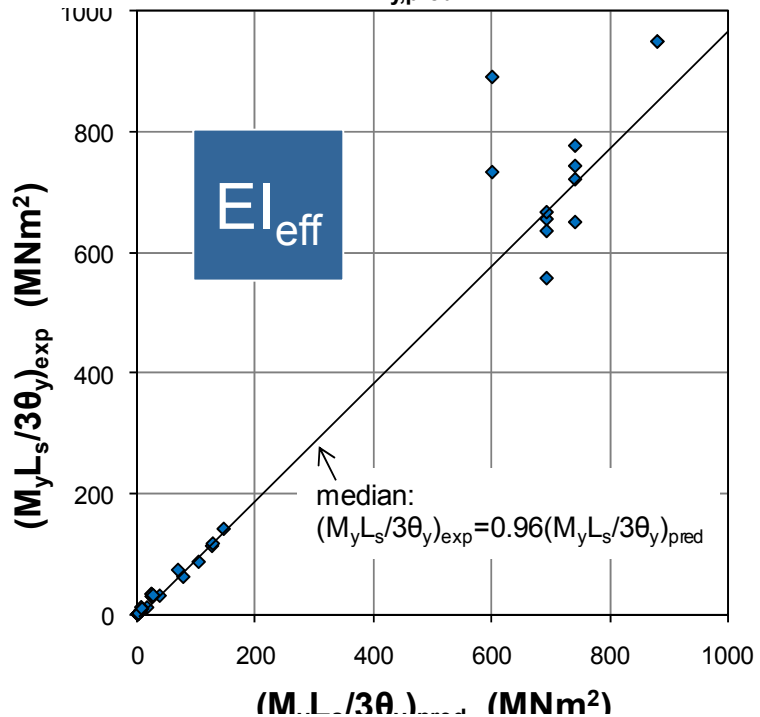
no.tests: 42
median=1.00
CoV=15.2%



no.tests: 42
median=0.98
CoV=22%



no.tests: 42
median=0.96
CoV=22.2%



Extension of empirical ultimate plastic chord rotation for rectangular members with lap-spliced bars & FRP-wrapping – Biskinis & Fardis 2007, 2013

- Required lapping for no adverse effect of lap-splice on ultimate deformation

$$l_{ou,\min} = \frac{d_b f_y}{\left(1.05 + 14.5\left(\frac{2}{n_{tension}}\right)^2 \left(\frac{a\rho f_u}{f_c}\right)_{f,eff}\right)\sqrt{f_c}}$$

with:

$$\left(\frac{a\rho f_u}{f_c}\right)_{f,eff} = a_f c_f \min\left[0.4; \frac{\rho_f f_{u,f}}{f_c}\right]\left(1 - 0.5\min\left[0.4; \frac{\rho_f f_{u,f}}{f_c}\right]\right)$$

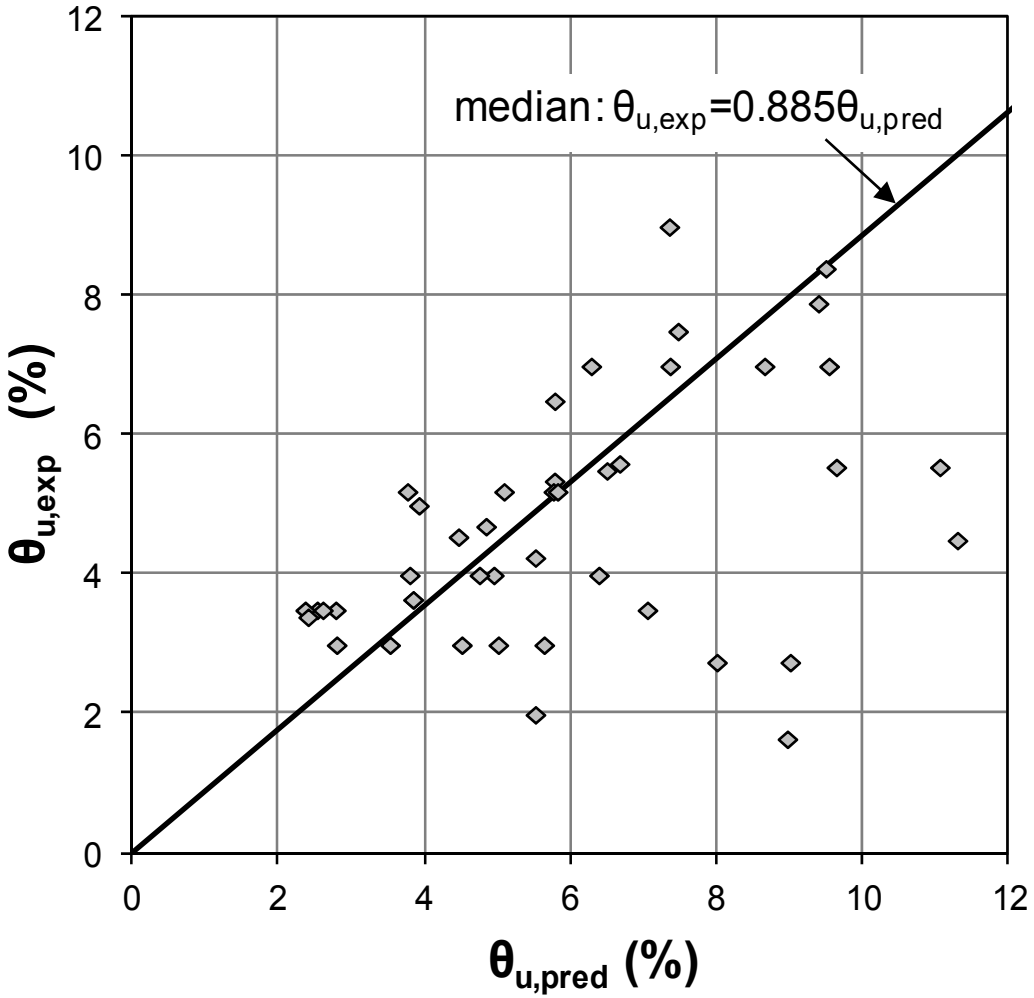
$f_{u,f}=E_f(k_{eff}\varepsilon_{u,f})$

$n_{tension}$: number of lapped bars on tension side of section

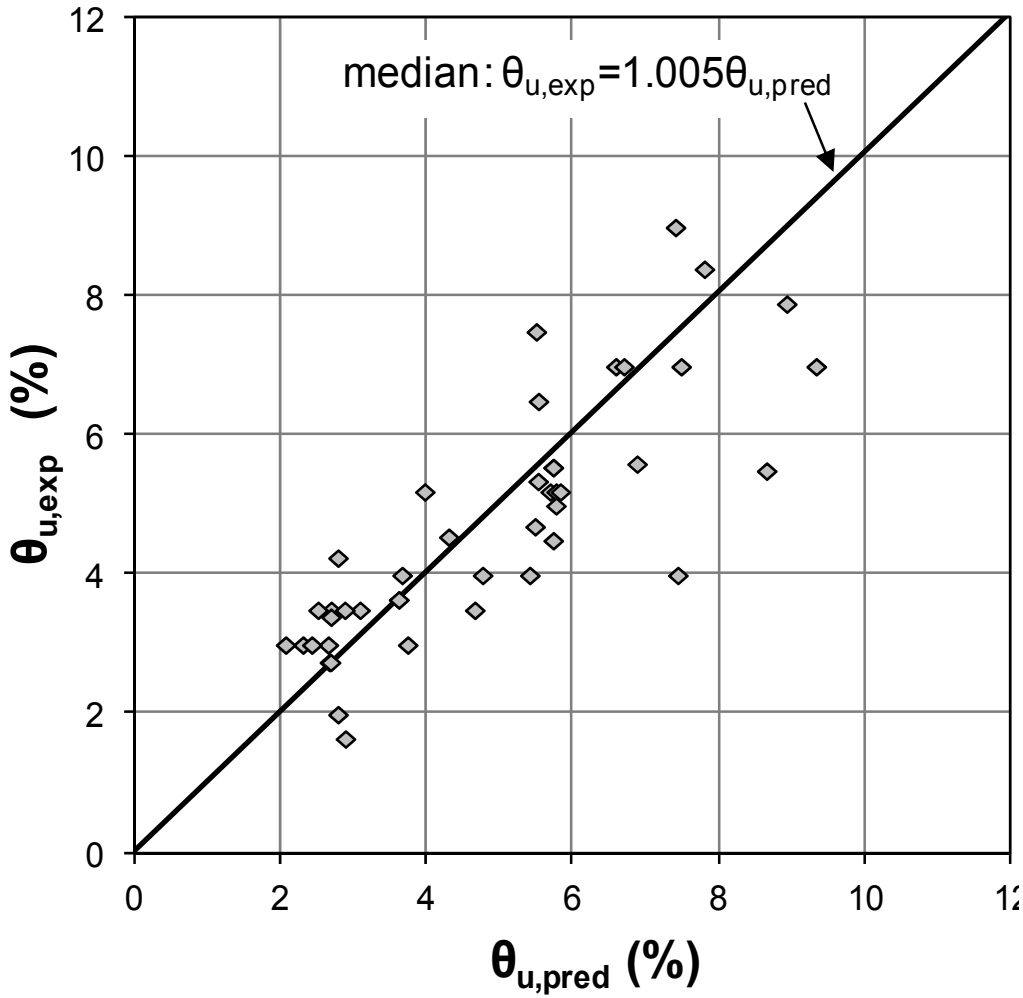
- If $l_o<l_{ou,min}$, θ_u^{pl} is reduced as: $\theta_{u,lap}^{pl} = \min\left(1, \frac{l_o}{l_{ou,\min}}\right)\theta_u^{pl}$
- $(\alpha\rho f_u/f_c)_{f,eff}$ neglected in θ_u^{pl} if $2/n_{tension}\leq 0.5$

Test-model comparison – ultimate cyclic chord rotation of FRP-wrapped of rectangular columns w/ lap-spliced bars, no.tests 44 (pre-damaged or not)
EN1998-3:2005 v Biskinis & Fardis 2007, 2013

EN 1998-3:2005
median=0.89, CoV=36.3%



Biskinis & Fardis 2013
median=1.005, CoV=23.2%



Extension of ultimate plastic chord rotation model with curvatures and plastic hinge length to circular columns with lap-spliced bars & FRP-wrapping – Biskinis & Fardis 2013

$$\theta_u = \theta_y + (\varphi_u - \varphi_y) L_{pl} \left(1 - 0.5 L_{pl} / L_s\right) + a_{sl} \Delta\theta_{u,slip}$$

$$\Delta\theta_{u,slip} = 5.5 d_{bL} \varphi_u$$

L_{pl}, f_{cc}, ε_{cu} as in members (FRP-wrapped or not) with continuous bars

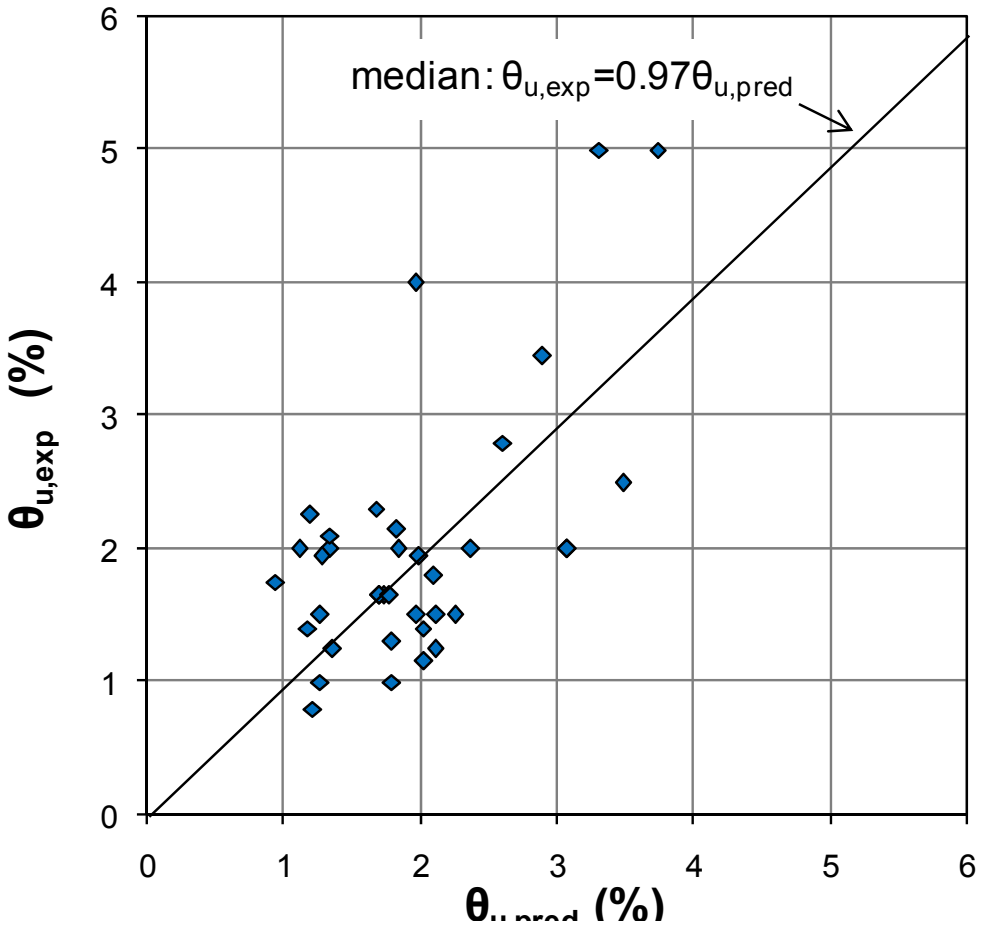
Steel strain at ultimate deformation of member depends on lap length, if $l_o < l_{ou,min}$:

$$\varepsilon_{su,l} = \left[1.2 \frac{l_o}{l_{ou,\min}} - 0.2\right] \varepsilon_{su} \geq \frac{l_o}{l_{oy,\min}} \frac{f_y}{E_s}$$

with:

$$l_{ou,min} = \frac{d_{bL} f_y}{\left(5/6 + 6 a \rho_{sh} f_{yh} / f_c\right) \sqrt{f_c}}$$

Test-to-prediction ratio, no.tests: 37
median=0.97, CoV=38.6%

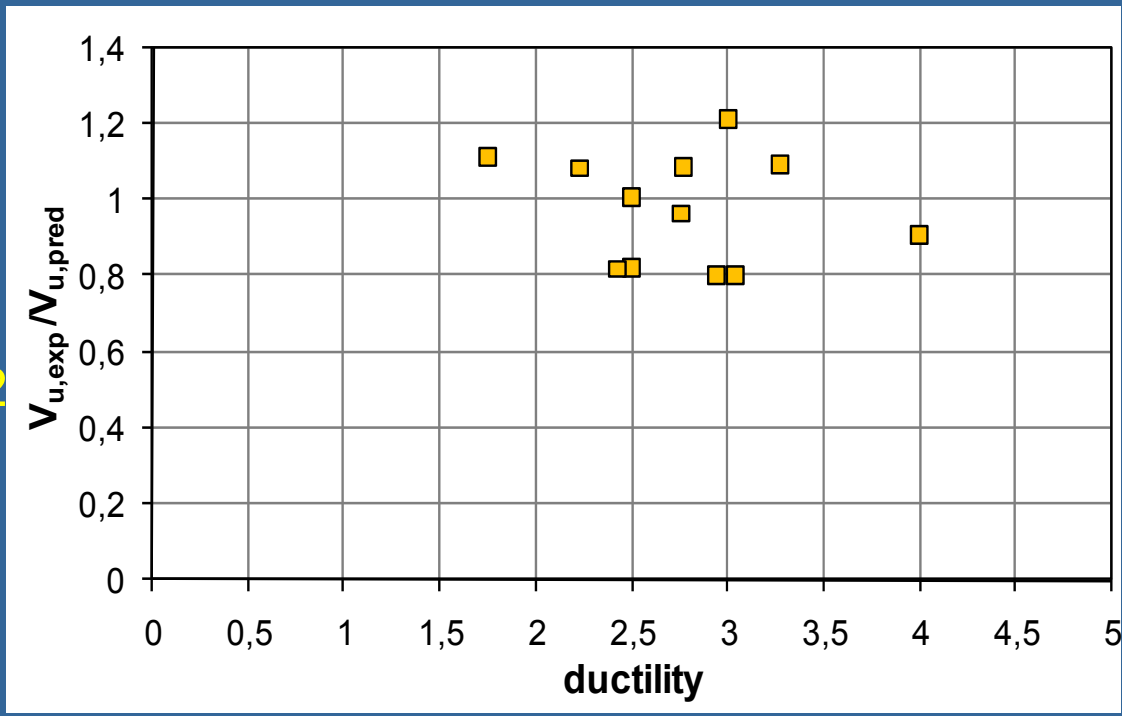


Cyclic shear resistance of FRP-wrapped rectangular members as controlled by diagonal tension, per EN1998-3:2005

$$V_R = \frac{h-x}{2L_s}\min\left(N, 0.55A_c f_c\right) + \left(1-0.05\min\left(5, \mu_\theta^{pl}\right)\right)\left[0.16\max(0.5, 100\rho_{tot})\left(1-0.16\min\left(5, \frac{L_s}{h}\right)\right)\sqrt{f_c}A_c + V_w\right] + V_f$$

- $V_f = \min(\varepsilon_{u,f}E_{u,f}, f_{u,f})\rho_f b_w z/2$: FRP-contribution to cyclic shear resistance:
  - ρ_f :FRP ratio, $\rho_f = 2t_f/b_w$;
  - $f_{u,f}$:FRP tensile strength.
- $V_w = \rho_w b_w z f_{yw}$: contribution of web steel (b_w: web width, z: int. lever arm; ρ_w: steel ratio)
- ρ_{tot}: total longitudinal steel ratio
- h: section depth
- x : depth of compression zone
- $A_c = b_w d$

Test-to-prediction ratio vs μ, no. tests 12
median=0.99, CoV=14.8%:



- In a FRP-retrofitted member the shear resistance as controlled by diagonal tension cannot exceed the shear resistance of old member as controlled by web crushing

RC jackets per EN1998-3:2005

Concrete Jackets (continued/anchored in joint; w/ or w/o lap splices in old member)

Calculation assumptions:

- Full composite action of jacket & old concrete assumed (jacketed member: monolithic"), even for minimal shear connection at interface (roughened interface, steel dowels epoxied into old concrete: useful but not essential);
- f_c of "monolithic member"= that of the jacket (avoid large differences in old & new f_c)
- Axial load considered to act on full, composite section;
- Longitudinal reinforcement of jacketed column: mainly that of the jacket. Vertical bars of old column considered at actual location between tension & compression bars of composite member (~ "web" longitudinal reinforcement), with its own f_y;
- Only the transverse reinforcement of the jacket is considered for confinement;
- For shear resistance, the old transverse reinforcement taken into account only in walls, if anchored in the (new) boundary elements
- The detailing & any lap-splicing of jacket reinforcement are taken into account.

Then:

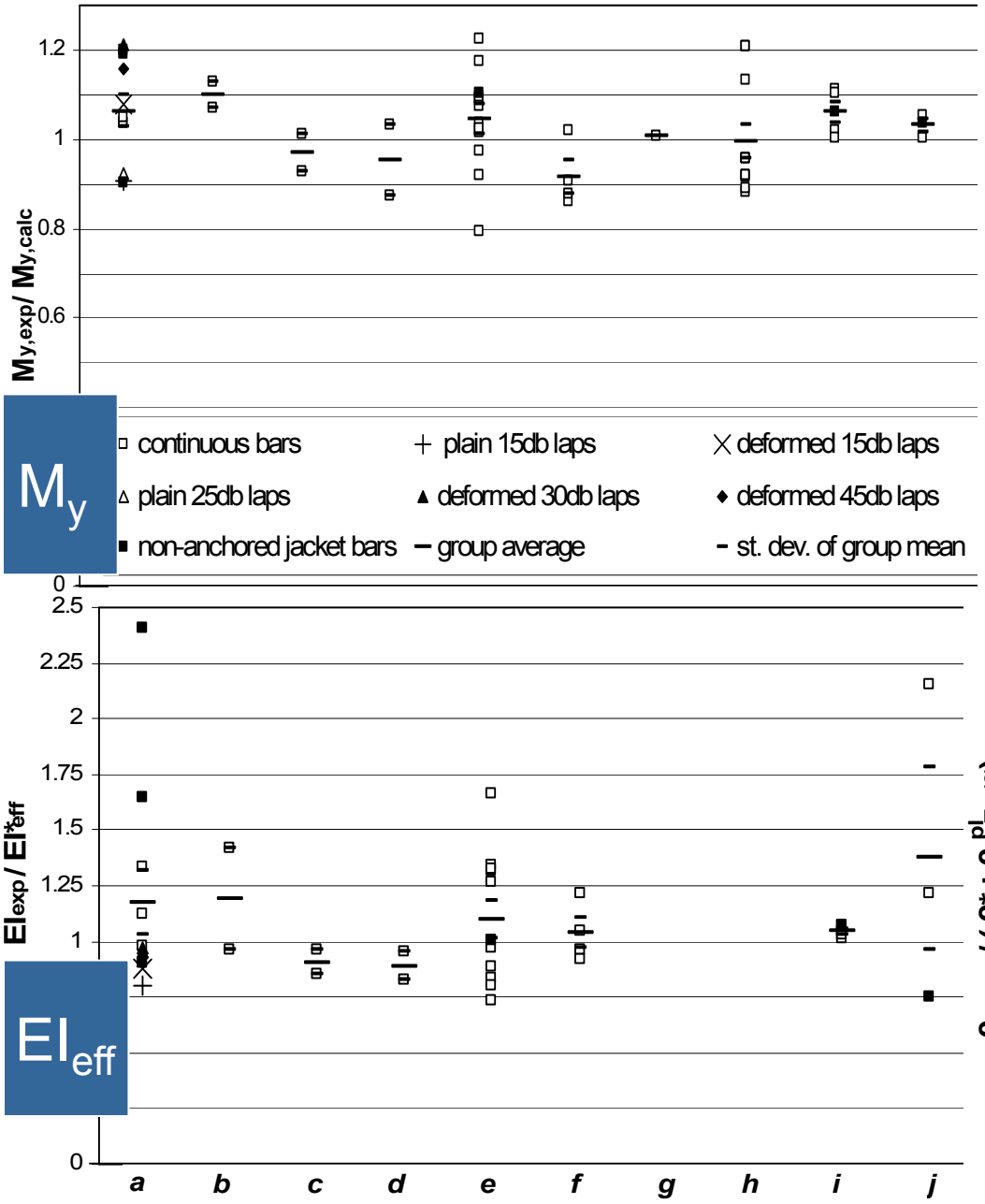
- ✓ M_R & M_y of jacketed member: ~100% of
- ✓ θ_y of jacketed member for pre-yield (elastic) stiffness: ~105% of
- ✓ Shear resistance of jacketed member: ~100% of
- ✓ Flexure-controlled ultimate deformation θ_u: ~100% of

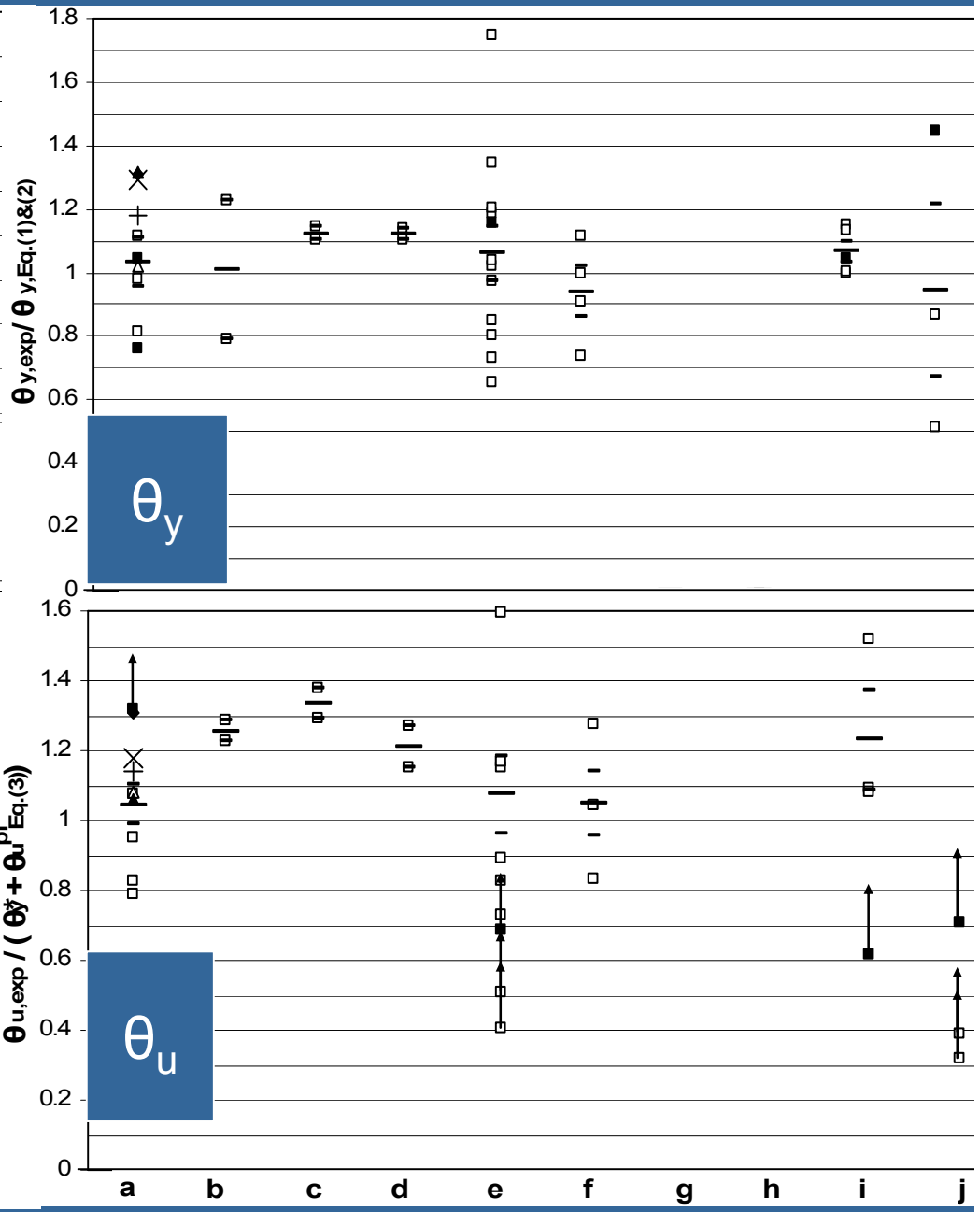
those of "monolithic member" calculated w/ assumptions above.

If jacket bars are not continued/anchored in the joint:

The jacket is considered only to confine fully the old column section.

54 jacketed members w/ or w/o lap splices: test-to-calculated as monolithic





a: no treatment, **b**: no treatment, predamage, **c**: welded U-bars, **d**: dowels, **e**: roughened, **f**: roughened / predamage, **g**: U-bars+ roughened, **h**: U-bars+roughened / predamage, **i**: roughened+dowels, **j**: roughened+dowels / predamage

Steel jackets per EN1998-3:2005

Steel Jackets (not continued/anchored in joint)

Jacket stops before the joint (several mm gap to joint face)

- Flexural resistance, pre-yield (elastic) stiffness & flexure-controlled ultimate deformation of RC member is not enhanced by jacket (flexural deformation capacity ~same as in old member inside jacket, no benefit from confinement);
- 50% of shear resistance of steel jacket, $V_j = A_j f_{yj} h$, can be relied upon for shear resistance of retrofitted member (suppression of shear failure before or after flexural yielding);
- Lap-splice clamping via friction mechanism at jacket-member interface, if jacket extends to ~1.5 times splice length and is bolt-anchored to member at end of splice region & ~1/3 its height from the joint face (anchor bolts at third-point of side)