MODELLING THE BEHAVIOR OF CONCRETE MEMBERS:
DEVELOPMENTS SINCE THE COMPLETION OF EN 1998-3:2005

Michael N. Fardis
University of Patras
Effective elastic stiffness: secant-to-yielding

Member deformation:
- Chord-rotation
- Section deformation: curvature
Practical expressions for the yield & failure properties developed for EN 1998-3:2005 - their advancement since 2005

M.N.FARDIS & D.BISKINIS, Deformation Capacity of RC Members, as Controlled by Flexure or Shear. Proceedings of International Symposium on Performance-based Engineering for Earthquake Resistant Structures honoring Prof. Shunsuke Otani University of Tokyo, Sept. 2003, pp. 511-530.


D.BISKINIS, M.N.FARDIS, Effect of Lap Splices on Flexural Resistance and Cyclic Deformation Capacity of RC Members, Beton & Stahlbetonbau, 102, 2007

D.BISKINIS & M.N.FARDIS, Cyclic Deformation Capacity of FRP-Wrapped RC Columns or Piers, with Continuous or Lap-Spliced Bars, 8th International Symposium on Fiber Reinforced Polymer Reinforcement for Concrete Structures (FRPRCS-8), Patras, July 2007.


D.BISKINIS & M.N.FARDIS, Cyclic Deformation Capacity, Resistance and Effective Stiffness of RC Members with or without Retrofitting, 14th World Conference on Earthquake Engineering, Beijing, paper 05-03-0153, Oct. 2008.


D.BISKINIS & M.N.FARDIS, Flexure-Controlled Ultimate Deformations of Members with Continuous or Lap-Spliced Bars, Structural Concrete, Vol. 11, No. 2, June 2010, 93-108.

D.BISKINIS & M.N.FARDIS, Deformations at Flexural Yielding of Members with Continuous or Lap-Spliced Bars, Structural Concrete, Vol. 11, No. 3, September 2010, 127-138.


## Experimental Database

1. Range/mean of parameters in tests for calibration of expressions for the member chord rotation and secant stiffness at yielding

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1653 rectangular beams/columns</th>
<th>214 rectangular walls</th>
<th>229 members of non-rectangular section</th>
<th>307 circular columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>Section depth or diameter, $h$ (m)</td>
<td>min/max</td>
<td>mean</td>
<td>min/max</td>
<td>mean</td>
</tr>
<tr>
<td></td>
<td>0.1 / 2.4</td>
<td>0.31</td>
<td>0.4 / 3.0</td>
<td>1.19</td>
</tr>
<tr>
<td>Shear-span-to-depth ratio, $L_s/h$</td>
<td>1 / 13.3</td>
<td>3.78</td>
<td>0.45 / 5.53</td>
<td>1.92</td>
</tr>
<tr>
<td>Section aspect ratio, $h/b_w$</td>
<td>0.2 / 4</td>
<td>1.3</td>
<td>4 / 30</td>
<td>10.9</td>
</tr>
<tr>
<td>$f_c$ (MPa)</td>
<td>9.6 / 175</td>
<td>37.7</td>
<td>13.5 / 109</td>
<td>35.8</td>
</tr>
<tr>
<td>Axial-load-ratio, $N/A_c f_c$</td>
<td>-0.05 / 0.9</td>
<td>0.126</td>
<td>0 / 0.86</td>
<td>0.10</td>
</tr>
<tr>
<td>Transverse steel ratio, $\rho_w$ (%)</td>
<td>0 / 3.54</td>
<td>0.62</td>
<td>0.05 / 2.18</td>
<td>0.54</td>
</tr>
<tr>
<td>Total longitudinal steel ratio $\rho_{tot}$ (%)</td>
<td>0.2 / 8.55</td>
<td>1.97</td>
<td>0.07 / 4.27</td>
<td>1.5</td>
</tr>
<tr>
<td>Diagonal steel ratio, $\rho_d$ (%)</td>
<td>0 / 1.68</td>
<td>0.027</td>
<td>0 / 0.25</td>
<td>0.005</td>
</tr>
<tr>
<td>Transverse steel yield stress $f_{yw}$, MPa</td>
<td>118 / 2050</td>
<td>468</td>
<td>220 / 1375</td>
<td>443</td>
</tr>
<tr>
<td>Longitudinal steel yield stress $f_y$, MPa</td>
<td>247 / 1200</td>
<td>440.2</td>
<td>276 / 1273</td>
<td>470</td>
</tr>
</tbody>
</table>
## Experimental Database

### 2. Range/mean of parameters in tests for calibration of expressions for ultimate curvature in monotonic or cyclic loading

<table>
<thead>
<tr>
<th>Parameter</th>
<th>415 rectangular beams/columns</th>
<th>59 rectangular walls</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>254 monotonic</td>
<td>160 cyclic</td>
</tr>
<tr>
<td>section depth or diameter, $h$ (m)</td>
<td>min/max</td>
<td>mean</td>
</tr>
<tr>
<td>section aspect ratio, $h/b_w$</td>
<td>0.12 / 0.8</td>
<td>0.31</td>
</tr>
<tr>
<td>$f_c$ (MPa)</td>
<td>19.7 / 99.4</td>
<td>34.8</td>
</tr>
<tr>
<td>axial-load-ratio, $N/Af_c$</td>
<td>0 / 0.78</td>
<td>0.08</td>
</tr>
<tr>
<td>transverse steel ratio, $\rho_w$ (%)</td>
<td>0 / 2.38</td>
<td>0.345</td>
</tr>
<tr>
<td>total longitudinal steel ratio $\rho_{tot}$ (%)</td>
<td>0 / 3.68</td>
<td>1.4</td>
</tr>
<tr>
<td>transverse steel yield stress, $f_{yw}$, MPa</td>
<td>0 / 596</td>
<td>419</td>
</tr>
<tr>
<td>longitudinal steel yield stress, $f_y$, MPa</td>
<td>277 / 596</td>
<td>490</td>
</tr>
</tbody>
</table>
### 3. Range/mean of parameters in tests for calibration of expressions for the member ultimate cyclic chord rotation

<table>
<thead>
<tr>
<th>Parameter</th>
<th>1159 rectangular beams/columns</th>
<th>95 rectangular walls</th>
<th>53 members of non-rectangular section</th>
<th>143 circular columns</th>
</tr>
</thead>
<tbody>
<tr>
<td>section depth or diameter, $h$ (m)</td>
<td>0.1 / 2.4</td>
<td>0.4 / 2.75</td>
<td>0.2 / 3.4</td>
<td>0.2 / 1.83</td>
</tr>
<tr>
<td>shear-span-to-depth ratio, $L_s/h$</td>
<td>1 / 13.3</td>
<td>0.5 / 5.53</td>
<td>0.65 / 8.33</td>
<td>1.77 / 10</td>
</tr>
<tr>
<td>section aspect ratio, $h/b_w$</td>
<td>0.2 / 6</td>
<td>2.5 / 28.3</td>
<td>2.5 / 36</td>
<td>1.0</td>
</tr>
<tr>
<td>$f_c$ (MPa)</td>
<td>12.2 / 175</td>
<td>13.5 / 109</td>
<td>20.8 / 83.6</td>
<td>23.1 / 90</td>
</tr>
<tr>
<td>axial-load-ratio, $N/Af_c$</td>
<td>-0.1 / 0.9</td>
<td>0.05 / 2.18</td>
<td>0.2 / 6.19</td>
<td>0.1 / 8.83</td>
</tr>
<tr>
<td>transverse steel ratio, $\rho_w$ (%)</td>
<td>0.015 / 3.37</td>
<td>0.07 / 4.27</td>
<td>0.2 / 6.19</td>
<td>0.75 / 5.5</td>
</tr>
<tr>
<td>total longitudinal steel ratio $\rho_{\text{tot}}$ (%)</td>
<td>0 / 6.29</td>
<td>0.04 / 2.09</td>
<td>0.2 / 6.19</td>
<td>0.75 / 5.5</td>
</tr>
<tr>
<td>diagonal steel ratio, $\rho_d$ (%)</td>
<td>0 / 1.68</td>
<td>0.004</td>
<td>0 / 0.25</td>
<td>0 / 0.25</td>
</tr>
<tr>
<td>transverse steel yield stress, $f_{yw}$, MPa</td>
<td>118 / 1497</td>
<td>220 / 1375</td>
<td>178 / 1375</td>
<td>200 / 1569</td>
</tr>
<tr>
<td>longitudinal steel yield stress, $f_y$, MPa</td>
<td>281 / 1275</td>
<td>276 / 1273</td>
<td>331 / 596</td>
<td>240 / 648</td>
</tr>
</tbody>
</table>
Yield & failure properties of RC sections
\( M-\varphi \) at yielding of section w\/ rectangular compression zone (width \( b \), effective depth \( d \)) – section analysis

- **Yield moment (from moment-equilibrium & elastic \( \sigma-\varepsilon \) laws):**

\[
\frac{M_y}{bd^3} = \varphi_y \left[ \frac{E_c}{2} \left( \frac{\xi_y^2}{2} \left( 0.5 (1 + \delta_1) \right) - \frac{\xi_y}{3} \right) + \frac{E_s}{2} \left[ (1 - \xi_y) \rho_1 + \left( \xi_y - \delta_1 \right) \rho_2 + \frac{\rho_v}{6} (1 - \delta_1) \right] \right] \left( 1 - \delta_1 \right)
\]

- \( \rho_1, \rho_2 \) : tension & compression reinforcement ratios, \( \rho_v \) : “web” reinforcement ratio, ~uniformly distributed between \( \rho_1, \rho_2 \) : (all normalized to \( bd \)); \( \delta_1 = d_1/d \).

- **Curvature at yielding of tension steel:**

\[
\varphi_y = \frac{f_y}{E_s \left( 1 - \xi_y \right) d}
\]

- from axial force-equilibrium & elastic \( \sigma-\varepsilon \) laws (\( \alpha = \frac{E_s}{E_c} \)):

\[
\xi_y = \left( \alpha^2 A^2 + 2 \alpha B \right)^{1/2} - \alpha A
\]

\[
A = \rho_1 + \rho_2 + \rho_v + \frac{N}{bdf_y}, \quad B = \rho_1 + \rho_2 \delta_1 + 0.5 \rho_v (1 + \delta_1) + \frac{N}{bdf_y}
\]

- **Curvature at ~onset of nonlinearity of concrete:**

\[
\varphi_y = \frac{\varepsilon_c}{\xi_y d} \approx \frac{1.8f_c}{E_c \xi_y d}
\]

\[
A = \rho_1 + \rho_2 + \rho_v - \frac{N}{\varepsilon_c E_s bd} \approx \rho_1 + \rho_2 + \rho_v - \frac{N}{1.8 \alpha bdf_c}, \quad B = \rho_1 + \rho_2 \delta_1 + 0.5 \rho_v (1 + \delta_1)\]
Moment at corner of bilinear envelope to experimental moment-deformation curve vs yield moment from section analysis

Left: 2085 beams/columns, CoV:16.3%; Right: 224 rect. walls, CoV:16.9%

Bias by +2.5% or -1%, because corner of bilinear envelope of the experimental moment-deformation curve ≠ 1st yielding in section. Same bias considered to apply to predicted yield curvature.
Empirical formulas for yield curvature - section w/ rectangular compression zone

for beams or columns:

\[ \phi_y \approx \frac{1.54 f_y}{E_s d} \quad \phi_y \approx \frac{1.75 f_y}{E_s h} \]

for walls:

\[ \phi_y \approx \frac{1.37 f_y}{E_s d} \quad \phi_y \approx \frac{1.47 f_y}{E_s h} \]

Empirical expressions don’t have a bias w.r.to experimental yield moment; but scatter is larger:

In ~2100 test beams, columns or walls: CoV: ~18%
Flexural failures - columns
Flexural failures - beams
Conventional definition of ultimate deformation

The value beyond which, any increase in deformation cannot increase the resistance above 80% of the maximum previous (ultimate) resistance.
Ultimate curvature of section with rectangular compression zone, from section analysis

- **Concrete σ-ε law:**
  - Parabolic up to $f_c$, $\varepsilon_{co}$,
  - Constant stress (rectangular) for $\varepsilon_{co} < \varepsilon < \varepsilon_{cu}$

- **Steel σ-ε law:**
  - Elastic-perfectly plastic, if steel strain rather low and concrete fails first;
  - Elastic-linearly strain-hardening, if steel strains are relatively high and steel breaks at stress and strain $f_t$, $\varepsilon_{su}$. 
Possibilities for ultimate curvature:

1. Section fails by rupture of tension steel, $\varepsilon_{s1} = \varepsilon_{su}$, before extreme compression fibers reach their ultimate strain (spalling), $\varepsilon_c < \varepsilon_{cu} \rightarrow$
   
   Ultimate curvature occurs in unspalled section, due to steel rupture: 
   $$\varphi_{su} = \frac{\varepsilon_{su}}{(1 - \xi_{su})d}$$

2. Compression fibres reach their ultimate strain (spalling): $\varepsilon_c = \varepsilon_{cu} \rightarrow$
   the confined concrete core becomes now the member section.

   Two possibilities:

   i. The moment capacity of the spalled section, $M_{Ro}$, never increases above 80% of the moment at spalling, $M_{Rc}$: $M_{Ro} < 0.8M_{Rc} \rightarrow$
      
      Ultimate curvature occurs in unspalled section, due to the concrete: 
      $$\varphi_{cu} = \frac{\varepsilon_{cu}}{\xi_{cu}d}$$

   ii. Moment capacity of spalled section increases above 80% of the moment at spalling: $M_{Ro} > 0.8M_{Rc} \rightarrow$
      
      The confined concrete core is now the member section and Cases 1 and 2(i) - applied for the confined core - are the two possibilities for attainment of the ultimate curvature $\rightarrow \varphi_{su}, \varphi_{cu}$ calculated as above but for the confined core; the minimum of the two is the ultimate curvature.

   $$M_{Rc} = bd^2 f_c \left[ \frac{(1 - \delta_1)(\omega_1 + \omega_2)}{2} + \frac{\omega_v}{1 - \delta_1} \left( \xi - \delta_1 \right)(1 - \xi) - \frac{1}{3} \left( \frac{\xi_{fy}}{E_s\varepsilon_{cu}} \right)^2 \right] + \xi \left[ \frac{1 - \xi}{2} - \frac{\varepsilon_{co}}{3\varepsilon_{cu}} \left( \frac{1}{2} - \xi + \frac{\varepsilon_{co}}{4\varepsilon_{cu}} \xi \right) \right]$$
Ultimate curvature of section w/ rectangular compression zone for steel rupture:

\[
\varphi_{su} = \frac{\varepsilon_{su}}{d(1 - \xi_{su})}
\]

- Steel ruptures before concrete crushes, after compression steel yields, if \( v \):

\[
\frac{\delta_1 \varepsilon_{su} + \varepsilon_y - (1 - \delta_1) \varepsilon_{co}}{3} + \omega_2 - \omega_1 \frac{f_t}{f_y} - \omega_1 \left(1 + \frac{f_t}{f_y}\right) \frac{\varepsilon_{su} - \varepsilon_y}{2(\varepsilon_{su} + \varepsilon_y)} \leq v \leq \frac{\varepsilon_{cu} - \varepsilon_{co}}{3} + \omega_2 - \omega_1 \frac{f_t}{f_y} - \omega_1 \left(1 + \frac{f_t}{f_y}\right) \left(\frac{1 + \delta_1}{(1 - \delta_1) \varepsilon_{su}} - \frac{1 - \delta_1}{\varepsilon_{cu}}\right)
\]

- \( \xi_{su} \) calculated from axial force equilibrium for:

\[
\xi_{su} = \frac{\left(1 - \delta_1\right) \left(1 + \frac{\varepsilon_{co}}{3\varepsilon_{su}}\right) + \left(1 + \frac{f_t}{f_y}\right) \omega_v}{\left(1 - \delta_1\right) \left(1 + \frac{\varepsilon_{co}}{3\varepsilon_{su}}\right) + \left(1 + \frac{f_t}{f_y}\right) \omega_v}
\]

\( v = \frac{N}{bd f_c}, \omega_1, \omega_2 : \) tension & compression mech. reinforcement ratios, \( \omega v: \) “web” mech. reinforcement ratio ~uniform distribution between \( \omega_1, \omega_2 \) \((\omega = \rho f_y / f_c); \delta_1 = d_1 / d\).

- Steel ruptures before concrete crushes or compression steel yields, if \( v \):

\[
v \leq \frac{\delta_1 \varepsilon_{su} + \varepsilon_y - (1 - \delta_1) \varepsilon_{co}}{3} + \omega_2 - \omega_1 \frac{f_t}{f_y} - \omega_1 \left(1 + \frac{f_t}{f_y}\right) \frac{\varepsilon_{su} - \varepsilon_y}{2(\varepsilon_{su} + \varepsilon_y)}
\]

- \( \xi_{su} \) from:

\[
\xi^2 - \left[1 + \frac{2 \varepsilon_{co}}{3\varepsilon_{su}} + \omega_1 \frac{f_t}{f_y} + \frac{\omega_v}{(1 - \delta_1)} \left(1 + \frac{f_t}{f_y}\right) \frac{\varepsilon_{su} - \delta_1 \varepsilon_y}{\varepsilon_y} \right] = 0
\]
Ultimate curvature of section w/ rectangular compression zone for concrete crushing:

\[ \varphi_{cu} = \frac{\varepsilon_{cu}}{\xi_{cu} d} \]

- Concrete crushes after tension steel yields, w/o compression steel yielding, if \( \nu \):

\[ \nu \leq \omega_2 - \omega_1 + \omega_v \left( \frac{\varepsilon_{cu} + \varepsilon_y}{1 - \delta_1 \left( \frac{\varepsilon_{cu} + \varepsilon_y}{\varepsilon_{cu} - \varepsilon_y} - 1 \right)} + \delta_1 \frac{\varepsilon_{cu} - \varepsilon_{co}}{3 \left( \varepsilon_{cu} - \varepsilon_y \right)} \right) \]

- \( \xi_{cu} \) from axial force equilibrium, for:

\[ 1 - \frac{\varepsilon_{co}}{3 \varepsilon_{cu}} + \frac{\omega_v}{2(1 - \delta_1)} \left( \frac{\varepsilon_{cu} + \varepsilon_y}{\varepsilon_{cu} \varepsilon_y} \right)^2 \xi^2 - \left[ \nu + \omega_1 - \omega_2 \frac{\varepsilon_{cu}}{\varepsilon_y} + \frac{\omega_v}{2(1 - \delta_1)} \left( 1 + \frac{\varepsilon_{cu} \delta_1}{\varepsilon_y} \right) \right] \xi - \left[ \omega_2 - \frac{\omega_v \delta_1}{2(1 - \delta_1)} \right] \frac{\varepsilon_{cu} \delta_1}{\varepsilon_y} = 0 \]

- Concrete crushes w/ tension & compression steel yielding, if \( \nu \):

\[ \omega_2 - \omega_1 + \frac{\omega_v}{1 - \delta_1} \left( \frac{\varepsilon_{cu} + \varepsilon_y}{\varepsilon_{cu} - \varepsilon_y} - 1 \right) + \delta_1 \frac{\varepsilon_{cu} - \varepsilon_{co}}{3 \left( \varepsilon_{cu} - \varepsilon_y \right)} \leq \nu < \omega_2 - \omega_1 + \frac{\omega_v}{1 - \delta_1} \left( \frac{\varepsilon_{cu} - \varepsilon_y}{\varepsilon_{cu} + \varepsilon_y} - \delta_1 \right) + \frac{\varepsilon_{cu} - \varepsilon_{co}}{3 \varepsilon_{cu} + \varepsilon_y} \]

- \( \xi_{cu} \) from:

\[ \xi_{cu} = \frac{(1 - \delta_1)(\nu + \omega_1 - \omega_2) + (1 + \delta_1)\omega_v}{(1 - \delta_1) \left( 1 - \frac{\varepsilon_{co}}{3 \varepsilon_{cu}} \right) + 2 \omega_v} \]

- Concrete crushes after compression steel yields, w/o tension steel yielding, if \( \nu \):

- \( \xi_{cu} \) from:

\[ \omega_2 - \omega_1 + \frac{\omega_v}{1 - \delta_1} \left( \frac{\varepsilon_{cu} - \varepsilon_y}{\varepsilon_{cu} + \varepsilon_y} - \delta_1 \right) + \frac{\varepsilon_{cu} - \varepsilon_{co}}{3 \left( \varepsilon_{cu} + \varepsilon_y \right)} \leq \nu \]

\[ 1 - \frac{\varepsilon_{co}}{3 \varepsilon_{cu}} - \frac{\omega_v}{2(1 - \delta_1)} \left( \frac{\varepsilon_{cu} - \varepsilon_y}{\varepsilon_{cu} \varepsilon_y} \right)^2 \xi^2 + \left[ \omega_2 + \omega_1 \frac{\varepsilon_{cu}}{\varepsilon_y} - \nu + \frac{\omega_v}{1 - \delta_1} \left( \frac{\varepsilon_{cu}}{\varepsilon_y} - \delta_1 \right) \right] \xi - \left[ \omega_1 + \frac{\omega_v}{2(1 - \delta_1)} \right] \frac{\varepsilon_{cu}}{\varepsilon_y} = 0 \]
Confined core after spalling of concrete cover.
Parameters are denoted by an asterisk and computed with:
- $b, d, d_i$ replaced by geometric parameters of the core: $b_c, d_c, d_{c1}$;
- $N, \rho, \rho_c, \rho_{c1}$ normalized to $b_d$ instead of $bd$;
- $\sigma$-$\varepsilon$ parameters of confined concrete, $f_{cc}, \varepsilon_{cc}$, used in lieu of $f_c, \varepsilon_{cu}$.

Unconfined full section – Steel rupture

- $\varepsilon < \varepsilon_{c1}$?
  - yes: $\varepsilon_{cu}$ from Eq. (3.41)
  - no: $\varepsilon_{cu}$ from Eq. (3.40)

Unconfined full section – Spalling of concrete cover

- $\varepsilon < \varepsilon_{c1}$?
  - yes: $\varepsilon_{cu}$ from Eq. (3.47)
  - no: $\varepsilon_{cu}$ from Eq. (3.46)

Failure of compression zone (concrete)

- $\varepsilon < \varepsilon_{c2}$?
  - yes: $\varepsilon_{cu}$ from Eq. (3.49)
  - no: $\varepsilon_{cu}$ from Eq. (3.45)

Compute moment resistances:
- $M_{Rc}$ (of full, unspalled section) and
- $M_{Ro}$ (of confined core, after spalling of cover).

Ultimate curvature of confined core after spalling of concrete cover

Rupture of tension steel

- $\varepsilon < \varepsilon_{c1}$?
  - yes: $\varepsilon_{cu}$ from Eq. (3.41)
  - no: $\varepsilon_{cu}$ from Eq. (3.40)

Ultimate curvature of confined core after spalling of concrete cover

$M_{Ro} < 0.8M_{Rc}$?

- yes: $\varepsilon_{cu}$ from Eq. (3.37)
- no: $\varepsilon_{cu}$ from Eq. (3.45)
Steel: $\varepsilon_{su} = 2.5\%, 5\%, 6\%$ for steel class A, B, C per Eurocode 2

$$
\varepsilon_{cu,c} = 0.004 + 0.5\left(\alpha \rho_s f_{yw} / f_{cc}\right)
$$

**cyclic test results**

<table>
<thead>
<tr>
<th></th>
<th>Cyclic all</th>
<th>Concrete crushing</th>
<th>Steel rupture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>0.96</td>
<td>0.90</td>
<td>0.98</td>
</tr>
<tr>
<td>C.o.V.</td>
<td>46.7%</td>
<td>55.1%</td>
<td>38.0%</td>
</tr>
<tr>
<td>No.</td>
<td>205</td>
<td>97</td>
<td>108</td>
</tr>
</tbody>
</table>
Test results vs ultimate curvature w/ failure strains for cyclic flexure per Biskinis & Fardis 2010 (adopted in *fib* MC2010):

- **Monotonic flexure:**
  - max. available steel strain: $(7/12)\epsilon_{su}$

- **Cyclic flexure:**
  - max. available steel strain: $(3/8)\epsilon_{su}$

\[
\epsilon_{cu}^* = 0.0035 + \left( \frac{10}{h_c (mm)} \right)^2 + 0.285 \frac{\alpha \omega_w}{1 + K}
\]

\[
\epsilon_{cu}^* = 0.0035 + \left( \frac{10}{h_c (mm)} \right)^2 + 0.2 \frac{\alpha \omega_w}{1 + K}
\]

monotonic & cyclic data, no. 474, median=1.00, CoV=49.7%

cyclic test data:

<table>
<thead>
<tr>
<th></th>
<th>Cyclic all</th>
<th>Concrete crushing</th>
<th>Steel rapture</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>0.99</td>
<td>0.99</td>
<td>1.01</td>
</tr>
<tr>
<td>C.o.V.</td>
<td>44.2%</td>
<td>52.6%</td>
<td>34.2%</td>
</tr>
<tr>
<td>No.</td>
<td>205</td>
<td>97</td>
<td>108</td>
</tr>
</tbody>
</table>
Yield & failure properties of RC members
Flexural behavior at member level (Moment-chord rotation)

Definition of chord rotations, $\theta$, at member ends

$$\theta_A = \frac{1}{x_B - x_A} \int_{x_A}^{x_B} \varphi(x)(x_B - x)dx$$

$$\theta_B = \frac{1}{x_B - x_A} \int_{x_A}^{x_B} \varphi(x)(x_A - x)dx$$

Elastic moments at ends A, B from chord rotations at A, B:

- $M_A = (2EI/L)(2\theta_A + \theta_B)$,
- $M_B = (2EI/L)(2\theta_B + \theta_A)$
Fixed-end rotation of member end due to bar slippage from their anchorage zone beyond member end

- Slippage of tension bars from region beyond end section (e.g. from joint or footing) → rigid-body rotation of entire shear span = \text{fixed-end rotation}, \theta_{\text{slip}}

  (included in measured chord-rotations of test specimen w.r. to base or joint; doesn’t affect measured relative rotations between any two member sections).

- If \( s = \) slippage of tension bars from anchorage → \( \theta_{\text{slip}} = s/(1-\xi)d \)

- If bond stress uniform over straight length \( l_b \) of bar beyond section of maximum moment → bar stress decreases along \( l_b \) from \( \sigma_s = f_{yL} \) at yielding) at section of maximum \( M \) to zero at end of \( l_b \rightarrow s = \sigma_s l_b/(2E_s) \)

- \( l_b = \) bond force demand per unit length \( (=A_s \sigma_s/(\pi d_{bL})=d_{bL} \sigma_s /4) \), divided by ~bond strength (assume \( =\sqrt{f_c} \))

- \( \varepsilon_s = \sigma_s/E_s/ (1-\xi)d = \varphi \)

- At yielding of member end section

\[
\theta_{y,\text{slip}} = \frac{\varphi y d_{bL} f_y}{8\sqrt{f_c}}
\]

(\( f_{yL}, f_c \) in MPa)
Fixed-end rotation of member end due to rebar pull-out from anchorage zone beyond member end, at member yielding

\( \frac{\varphi_{y,\text{measured}}}{(\varphi_{y,\text{predicted}} + \theta_{y,\text{slip}}/l_{\text{gauge}})} \) no.160 measurements w/ slip:
median = 1.0, C.o.V = 34%

Ratio:
experimental-to-predicted yield curvature (w/ correction for fixed-end rotation) in terms of gauge length
Chord rotation of shear span at yielding of end section
per EN 1998-3:2005

- Rect. beams or columns:
  \[
  \theta_y = \varphi_y \frac{L_s + a_V z}{3} + 0.0014 \left(1 + 1.5 \frac{h}{L_s}\right) + a_{sl} \theta_{y,\text{slip}}
  \]

- Rect. or non-rect. walls:
  \[
  \theta_y = \varphi_y \frac{L_s + a_V z}{3} + 0.0013 + a_{sl} \theta_{y,\text{slip}}
  \]

“Shift rule”:
Diagonal cracking shifts value of force in tension reinforcement to a section at a distance from member end equal to \(z\) (internal lever arm).

- \(z = d - d_1\) in beams, columns, or walls of barbelled or T-section,
- \(z = 0.8h\) in rectangular walls.

\(a_v = 0\), if \(V_{Rc} > M_y/L_s\);
\(a_v = 1\), if \(V_{Rc} \leq M_y/L_s\).

\(V_{Rc}\) = force at diagonal cracking per Eurocode 2
(in kN, dimensions in m, \(f_c\) in MPa):

\[
V_{Rc} = \max \left\{ 180 (100 \rho_1)^{1/3}, \ 35 \sqrt{1 + \sqrt{\frac{0.2}{d} f_c^{1/6}}} \left[1 + \sqrt{\frac{0.2}{d} f_c^{1/6}}\right] f_c^{1/3} + 0.15 \frac{N}{A_c} \right\} b_w d
\]

- \(a_{sl} = 0\), if no slip from anchorage zone beyond end section;
- \(a_{sl} = 1\), if there is slip from anchorage zone beyond end section.
Test-model comparison – $\theta_y$

Beams/rect. columns, no. tests: 1653

Median: $\theta_{y,\text{exp}} = 1.01\theta_{y,\text{pred}}$

Median $= 1.01$, CoV $= 32.1\%$
Test-model comparison – $\theta_y$
Walls, no. tests: 386

Expression in EN 1998-3:2005
median=0.97, CoV=31.1%

Modified expression adopted in MC2010

$$\theta_y = \phi_y \frac{L_s + a_y z}{3} + 0.00045 \left( 1 + \frac{5h}{3L_s} \right) + a_{sl} \theta_{y,sl}$$
median=1.01, CoV=30.9%
Test-model comparisons - $\theta_y$

$$\theta_y = \varphi_y \frac{L_s + a V z}{3} + 0.0027 \left(1 - \min \left(1; \frac{2L_s}{15D}\right)\right) + a_{sl} \theta_{y,slip}$$

median=1.00, CoV=31.7%
Effective elastic stiffness, $EI_{eff}$ (for linear or nonlinear analysis)

- Part 1 of EC8 (for design of new buildings):
  - $EI_{eff}$: secant stiffness at yielding = 50% of uncracked gross-section stiffness.
  - OK in force-based design of new buildings (safe-sided for forces);
  - Unsafe in displacement-based assessment for displacement demands).

- More realistic:
  - Secant stiffness at yielding of end of shear span $L_s = M/V$
Test-model comparison – $EI_{\text{eff}}$

Beams/rect. columns, no. tests: 1616

median = 1.00, CoV = 32.1%

$$EI_{\text{eff}} = \frac{M_y L_s}{3 \theta_y}$$

medial:

$$(M_y L_s/3\theta_y)_{\text{exp}} = (M_y L_s/3\theta_y)_{\text{pred}}$$
Test-model comparison – $EI_{eff}$ Walls, no. tests: 386

$EI_{eff} = \frac{M_y L_s}{3\theta_y}$

With expression for $\theta_y$ in EN 1998-3:2005
median=1.035
CoV=42.8%

With new expression for $\theta_y$ of walls (adopted in MC2010)
median=0.98
CoV=41.1%
Test-model comparison – $EI_{\text{eff}}$

$$EI_{\text{eff}} = \frac{M_y L_s}{3\theta_y}$$

median=0.99, CoV=31.2%
Empirical secant stiffness to yielding, $EI_{\text{eff}}$, independent of amount of reinforcement - not in EN 1998-3:2005

$$\frac{EI_{\text{eff}}}{E_c I_c} = a \left( 0.8 + \ln \left( \max \left( \frac{L_s}{h} ; 0.6 \right) \right) \right) \left( 1 + 0.048 \min \left[ 50; \frac{N}{A_c} \right] \right)$$

(all variables known before dimensioning the longitudinal reinforcement)

- If there is slippage of longitudinal bars from their anchorage zone beyond the member end:
  - $a = 0.081$ for columns;
  - $a = 0.10$ for beams or non-rectangular walls (barbelled, T-, H-section);
  - $a = 0.115$ for rectangular walls;
  - $a = 0.12$ for members with circular section.
- If there is no slippage of longitudinal bars: effective stiffness $\times 4/3$
Test-model comparison – Empirical $EI_{\text{eff}}$, independent of amount of reinforcement - not in EN 1998-3:2005

Beams/columns
no. tests: 1616
median=1.00
CoV=36.1%

$\frac{M_y L_s}{3 \theta_y}$, exp $= EI_{\text{pred}}$
Test-model comparison —
Empirical $EI_{eff}$ independent of reinforcement, not in EN1998-3: 2005

Walls
no. tests: 386
median = 1.00
CoV = 44.6%

Circular columns
no. tests: 273
median = 0.995
CoV = 31.4%
Flexure-controlled ultimate chord rotation from curvature & plastic hinge length per EN 1998-3:2005

\[ \theta_u = \theta_y + \theta_{u}^{pl} = \theta_y + (\varphi_u - \varphi_y) L_{pl} \left( 1 - \frac{0.5 L_{pl}}{L_s} \right) \]

- \( \varphi_y \): yield curvature \( (\text{section analysis}) \);

**Option 1:** Confinement per Eurocode 2

\[ \alpha \rho_{sx} f_{yw} \leq 0.05 f_c : \quad f_{cc} = f_c + 5 \alpha \rho_{sx} f_{yw} \]
\[ \alpha \rho_{sx} f_{yw} > 0.05 f_c : \quad f_{cc} = 1.125 f_c + 2.5 \alpha \rho_{sx} f_{yw} \]
\[ \varepsilon_{cu,c} = \varepsilon_{co} + 0.2 \left( \alpha \rho_{sx} f_{yw} / f_c \right) \]
\[ L_{pl} = 0.1 L_s + 0.17 h + 0.24 d_b f_y / \sqrt{f_c} \]

- index \( c \): confined;
- \( \rho_s \): stirrup ratio;
- \( L_s = \text{M/V} \): shear span at member end;
- \( h \): section depth;
- \( d_b \): bar diameter;
- \( f_y, f_c \): MPa

**Option 2:** New, per EN 1998-3:2005

\[ f_{cc} = f_c \left[ 1 + 3.7 \left( \alpha \rho_{sx} f_{yw} / f_c \right)^{0.86} \right] \]
\[ \varepsilon_{cu,c} = 0.004 + 0.5 \left( \alpha \rho_{sx} f_{yw} / f_{cc} \right) \]
\[ L_{pl} = L_s / 30 + 0.2 h + 0.11 d_b f_y / \sqrt{f_c} \]

\( \alpha \): confinement effectiveness:
- rectangular section:
- circular section & hoops:

\[ \alpha = \left( 1 - \frac{s_h}{2b_c} \right) \left( 1 - \frac{s_h}{2h_c} \right) \left( 1 - \frac{\sum b_i^2}{6b_c h_c} \right) \]

- \( s_h \): centerline spacing of stirrups;
- \( D_c, b_c, h_c \): confined core dimensions to centerline of hoop;
- \( b_i \): centerline spacing along section perimeter of longitudinal bars (index: \( i \)) engaged by a stirrup corner or cross-tie.
Test-model comparison – ultimate chord rotation from curvatures & plastic hinge length per EN 1998-3:2005, no. tests: 1100

Option 1: confinement per EC2
median=0.88, CoV=52.3%

Option 2: new confinement per EC8-3
median=0.91, CoV=52.2%
Flexure-controlled ultimate chord rotation of rect. beams, columns, walls, non-rect. walls & circular columns from curvatures & plastic hinge length by Biskinis & Fardis 2010, 2013 - adopted in fibMC2010

Flexure-controlled ultimate chord rotation, accounting separately for slippage in yield-penetration length, from yielding till ultimate deformation:

$$\theta_u = \theta_y + a_{sl} \Delta \theta_{u,slip} + (\varphi_u - \varphi_y) L_{pl} \left(1 - \frac{L_{pl}}{2L_s}\right)$$

Confinement per fib MC2010:

$$f_{cc} = f_c \left[1 + 3.5 \left(\frac{\alpha \rho_w f_{yw}}{f_c}\right)^{3/4}\right]$$

- **Monotonic loading - Rect. beams, columns, walls, non-rectangular walls:**

$$\varepsilon_{cu,c} = 0.0035 + \left(\frac{10}{h_o (mm)}\right)^2 + 0.57 \frac{\alpha \rho_w f_{yw}}{f_{cc}}, \quad \varepsilon_{su,mon} = \frac{7}{12} \varepsilon_{su,normal}$$

$$L_{pl,mon} = h \left(1.1 + 0.04 \min \left(9; \frac{L_s}{h}\right)\right)$$

- **Cyclic loading:**

$$\varepsilon_{cu,c} = 0.0035 + \left(\frac{10}{h_o (mm)}\right)^2 + 0.4 \frac{\alpha \rho_w f_{yw}}{f_{cc}}, \quad \varepsilon_{su,cy} = \frac{3}{8} \varepsilon_{su,normal}$$

- **Rect. beams/columns/walls, non-rect. walls:**

$$L_{pl,cy} = 0.2h \left(1 + \frac{1}{3} \min \left(9; \frac{L_s}{h}\right)\right)$$

- **Circular columns:**

$$L_{pl,cy,cir} = 0.6D \left(1 + \frac{1}{6} \min \left(9; \frac{L_s}{D}\right)\right)$$
Post-yield fixed-end rotation of member end due to bar slippage from yield penetration length beyond member end, from yielding till ultimate flexural deformation per Biskinis & Fardis 2010, 2013 - adopted in fibMC2010 (not in EN 1998-3:2005)

- **Monotonic loading:**
  \[ \Delta \theta_{u,slip} = 9.5d_{bL} \varphi_u \quad \text{or} \quad = 16d_{bL} \left( \varphi_y + \varphi_u \right)/2 \]

- **Cyclic loading:**
  \[ \Delta \theta_{u,slip} = 5.5d_{bL} \varphi_u \quad \text{or} \quad = 10d_{bL} \left( \varphi_y + \varphi_u \right)/2 \]

Complete pull-out of beam bars, due to short anchorage in corner joint

← →
Test-model comparison – Cyclic ultimate chord rotation from curvatures & plastic hinge length per Biskinis & Fardis 2010, 2012

Rect. beams/columns/walls, non-rect. walls - no. tests: 1100
median=1.00, CoV=43.2%

Circular columns – no. tests: 143, median=1.00, CoV=30.3%

$$\theta_{um} = 0.016 \cdot (0.3^\nu) \left[ \frac{\max(0.01; \omega')}{\max(0.01; \omega)} \right] f_c \left( \frac{L_s}{h} \right)^{0.35} \left( \frac{a_{psx} f_{yw}}{f_c} \right)^{25} \left[ 1.25^{100 \rho_d} \right]$$

or:

$$\theta_{um}^{pl} = \theta_{um} - \theta_y = 0.0145 \cdot (0.25^\nu) \left[ \frac{\max(0.01; \omega')}{\max(0.01; \omega)} \right]^{0.3} f_c \left( \frac{L_s}{h} \right)^{0.35} \left( \frac{a_{psx} f_{yw}}{f_c} \right)^{25} \left[ 1.275^{100 \rho_d} \right]$$

- $\omega, \omega'$: mechanical ratio of tension (including web) & compression steel;
- $\nu$: $N/bh f_c$ ($b$: width of compression zone; $N>0$ for compression);
- $L_s/h$: $M/Vh$: shear span ratio;
- $\alpha$: confinement effectiveness factor:
  $$\alpha = \left( 1 - \frac{S_h}{2b} \right) \left( 1 - \frac{S_h}{2h_c} \right) \left( 1 - \frac{\sum b_i^2}{6b_c h_c} \right)$$
- $\rho_{sx}$: $A_{sh}/b_w s_h$: transverse steel ratio // direction of loading;
- $\rho_d$: ratio of diagonal reinforcement.

- Walls: 1st expression divided by 1.6; 2nd multiplied by 0.6
- Cold-worked brittle steel: 1st expression divided by 1.6; 2nd by 2.0

Non-seismically detailed members with continuous bars:
- Plastic part, $\theta_{um}^{pl} = \theta_{um} - \theta_y$, of ultimate chord rotation: divided by 1.2.
Test-model comparison – Empirical cyclic ultimate chord rotation of members with seismic detailing per EN 1998-3:2005 – no. tests 1100

Model for total $\theta_{um}$
median=1.00, CoV=37.8%

Model with $\theta_{um}=\theta_y+\theta_{pl}$
median=1.00, CoV=37.6%

Model for total $\theta_{um}$
median=1.00, CoV=30.6%

Model with $\theta_{um}=\theta_y+\theta_{pl}$
median=0.99, CoV=31.8%

\[
\theta_{u,pl} = \alpha_{st}^{hbw} (1 - 0.525\alpha_{cy})(1 + 0.6\alpha_{sl}) \left( 1 - 0.052\max\left(1.5; \min\left(10; \frac{h}{b_w}\right)\right) \right) (0.2)^{\frac{\max(0.01; \alpha_2)}{\max(0.01; \alpha_1)}} \min\left(9; \frac{L_s}{h}\right) \frac{1}{3} f_c^{0.25} \left( \frac{\alpha_p}{f_y}\right) 1.225^{100\rho_d}
\]

\(\alpha_{st}^{hbw}\):
- 0.022 for hot-rolled or Tempcore bars;
- 0.0095 for brittle cold-worked bars;

\(\alpha_{cy}\):
- = 1 for cyclic loading,
- = 0 for monotonic;

\(\alpha_{sl}\):
- = 1 if slippage of long. bars from anchorage zone is possible,
- = 0 otherwise;

\(b_w\): width of (one) web

Non-seismically detailed members:
- \(\theta_{u,um} = \theta_{um} - \theta_y\) divided by 1.2.

no. tests: 1100, median=1.00, CoV=37.6%
Effect of lap-splicing the column bars in the plastic hinge region
Members with ribbed bars lap-spliced over length $l_o$ inside the plastic hinge region per EN 1998-3:2005 or other options

- **EN 1998-3:2005:**
  1. Both bars in pair of lapped compression bars count as compression steel.
  2. For the yield properties $(M_y, \phi_y, \theta_y)$, the stress $f_s$ of tension bars is:

$$f_s = f_y(\frac{l_o}{l_{oy,\text{min}}}), \quad \text{if} \quad l_o < l_{oy,\text{min}} = (0.3 \frac{f_y}{\sqrt{f_c}})d_b \quad (f_y, f_c \text{ in MPa})$$

  3. Ultimate chord rotation

$$\theta_u = \theta_y + \theta^\text{pl}_u(\frac{l_o}{l_{ou,\text{min}}}), \quad \text{if} \quad l_o < l_{ou,\text{min}} = d_b f_y / [(1.05 + 14.5 \alpha_{rs} \omega_{sx}) \sqrt{\frac{f_c}{f_y}}]$$

- $f_y, f_c$ in MPa, $\omega_{sx} = \rho_{sx} f_y / f_c$: mech. transverse steel ratio // loading,
- $\alpha_{rs} = (1 - s_h / 2b_o) (1 - s_h / 2b_o) n_{\text{restr}} / n_{\text{tot}}$ (n_{\text{restr}} / n_{\text{tot}}: restrained-to-total lap-spliced bars).

- **Or, Eligeghausen & Lettow 2007 for fib MC2010:**

$$f_s = 51.2 \left( \frac{l_b}{d_b} \right)^{0.55} \left( \frac{f_c}{20} \right)^{0.25} \left( \frac{20}{\max(d_b; 20\text{mm})} \right)^{0.2} \left[ \left( \frac{c_d}{d_b} \right)^{1/3} \left( \frac{c_{\text{max}}}{c_d} \right)^{0.1} + k K_{tr} \right] \leq f_y, \quad k K_{tr} = \frac{1}{n_b d_b} \left( \frac{k_s n_1 A_{sh}}{s_h} \right)$$

- $c_d = \min[a/2; c_1; c] \geq d_b$, $c_d \leq 3d_b$
- $c_{\text{max}} = \max[a/2; c_1; c] \leq 5d_b$
- $f_y, f_c$ in MPa
Test-model comparison: Yield moment & chord rotation, effective stiffness, ultimate cyclic chord rotation - lapped ribbed bars per EN 1998-3:2005

- **\( M_y \)**: 114 tests, median = 1.00, CoV = 11.8%
- **\( \theta_y \)**: 92 tests, median = 1.05, CoV = 20%
- **\( E I_{\text{eff}} \)**: 92 tests, median = 0.955, CoV = 24.8%
- **\( \theta_u \)**: 81 tests, median = 1.035, CoV = 39.3%
Test-model comparison: Yield moment & chord rotation, effective stiffness, - lapped bars, steel stress per Eligehausen & Letow 2007

- Median = 0.98
- CoV = 12%

- Median = 1.03
- CoV = 20.5%

- Median = 0.97
- CoV = 24.6%
Members with **smooth hooked** bars, with or without lap splice (of length $l_o$) in the plastic hinge per EN 1998-3:2005

Provided that lapping $l_o \geq 15d_b$:

1. Yield properties $M_y, \phi_y, \theta_y$:
   - As in members with **continuous ribbed** bars.

2. Ultimate cyclic chord rotation:
   - For **continuous** bars:
     - $0.8\theta_{um} (\approx 0.95$ for smooth bars /1.2 for no seismic detailing) or
     - $\theta_{um} = \theta_y + 0.75\theta_{pl, um} (\approx 0.9$ for smooth bars /1.2 for no seismic detailing)
   - For **lapping** $l_o \geq 15d_b$:
     - $\theta_{um} \times 0.019(10 + \min(40, l_o/d_b))$ or
     - $\theta_{um} = \theta_y + \theta_{pl, um} \times 0.019 \min(40, l_o/d_b)$
   - $L_s$ for $\theta_{pl, um}$ not reduced by $l_o$

(ultimate condition not necessarily controlled by region right above the lap)

<table>
<thead>
<tr>
<th>Test-model-ratio for $\theta_{um}$</th>
<th>no laps</th>
<th>laps</th>
</tr>
</thead>
<tbody>
<tr>
<td>Median</td>
<td>1.015</td>
<td>1.03</td>
</tr>
<tr>
<td>C.o.V.</td>
<td>33.3%</td>
<td>33.4%</td>
</tr>
<tr>
<td>no. tests</td>
<td>34</td>
<td>11</td>
</tr>
</tbody>
</table>
Cyclic shear resistance of RC members
Cyclic shear resistance per EN 1998-3:2005

• Shear resistance in pl. hinge after flex. yielding, as controlled by stirrups
(linear decay of $V_c$ & $V_w$ with cyclic plastic rotation ductility ratio $\mu_{\theta_{pl}}=(\theta-\theta_y)/\theta_y>0$

\[
V_R = \frac{h-x}{2L_s} \min \left( N; 0.55 A_c f_c \right) + \left( 1 - 0.05 \min \left( 5; \mu_{\theta_{pl}} \right) \right) 0.16 \max \left( 0.5; 100 \rho_{\text{tot}} \right) \left( 1 - 0.16 \min \left[ 5; \frac{L_s}{h} \right] \right) \sqrt{f_c A_c + V_w}.
\]

$V_w = \rho_w b_w z f_{yw}$, due to stirrups ($b_w$: web width, $z$: internal lever arm; $\rho_w$: shear reinf. ratio)
$p_{\text{tot}}$: total longitudinal reinforcement ratio
$h$: section depth
$x$: depth of compression zone at yielding
$A_c = b_w d$

• Shear resistance as controlled by web crushing (diagonal compression)
  - Walls, before flexural yielding ($\mu_{\theta_{pl}} = 0$) or after (cyclic $\mu_{\theta_{pl}} > 0$):

\[
V_R = 0.85 \left( 1 - 0.06 \min \left( 5; \mu_{\theta_{pl}} \right) \right) \left( 1 + 1.8 \min \left( 0.15; \frac{N}{A_c f_c} \right) \right) \left( 1 + 0.25 \max (1.75, 100 \rho_{\text{tot}}) \right) \left( 1 - 0.2 \min \left( 2; \frac{L_s}{h} \right) \right) \sqrt{f_c b_w z}
\]

  - Squat columns ($L_s/h \leq 2$) after flexural yielding (cyclic $\mu_{\theta_{pl}} > 0$):

\[
V_R = \frac{4}{7} \left( 1 - 0.02 \min \left( 5; \mu_{\theta_{pl}} \right) \right) \left( 1 + 1.35 \frac{N}{A_c f_c} \right) \left( 1 + 0.45 \cdot 100 \rho_{\text{tot}} \right) \sqrt{\min \left( f_c, 40 \right) b_w z \sin 2\delta}
\]

$\delta$: angle between axis and diagonal of column ($\tan \delta = 0.5h/L_s$)
Test-model comparison: Cyclic shear resistance in plastic hinge (after flexural yielding) as controlled by stirrups, per EN 1998-3:2005

no. tests: 306
median=0.995
CoV=14.7%
Test-model comparison: Cyclic shear resistance as controlled by web crushing (diagonal compression), per EN 1998-3:2005

**Walls**

<table>
<thead>
<tr>
<th>$V_{\text{exp}}$ (kN)</th>
<th>$V_{\text{pred}}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>50</td>
<td>50</td>
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<tr>
<td>350</td>
<td>350</td>
</tr>
<tr>
<td>400</td>
<td>400</td>
</tr>
</tbody>
</table>

- no. tests: 62, median=1.00, CoV=19.3%

**Squat columns**

<table>
<thead>
<tr>
<th>$V_{\text{exp}}$ (kN)</th>
<th>$V_{\text{pred}}$ (kN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
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<td>350</td>
<td>350</td>
</tr>
<tr>
<td>400</td>
<td>400</td>
</tr>
</tbody>
</table>

- no. tests: 40, median=1.00, CoV=9.6%
FRP Jackets per EN1998-3:2005
### Experimental Database

#### 4. Range and mean values of parameters in the tests of FRP-jacketed rectangular columns in the database

<table>
<thead>
<tr>
<th>Parameter</th>
<th>219 columns with continuous bars (145 with CFRP, 24 with GFRP, 27 with AFRP and 23 with other composite)</th>
<th>45 columns with lap-spliced bars (42 with CFRP, 3 with GFRP)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>min-max</td>
<td>mean</td>
</tr>
<tr>
<td>effective depth, (d) (mm)</td>
<td>170-720</td>
<td>296</td>
</tr>
<tr>
<td>shear-span-to-depth ratio, (L_s/h)</td>
<td>1-7.4</td>
<td>3.55</td>
</tr>
<tr>
<td>concrete strength, (f_c) (MPa)</td>
<td>10.6-90</td>
<td>31.8</td>
</tr>
<tr>
<td>vertical bar yield stress, (f_y) (MPa)</td>
<td>295-816</td>
<td>431</td>
</tr>
<tr>
<td>stirrup yield stress, (f_{yw}) (MPa)</td>
<td>200-750</td>
<td>388</td>
</tr>
<tr>
<td>axial-load-ratio, (N/A_c f_c)</td>
<td>0-0.85</td>
<td>0.255</td>
</tr>
<tr>
<td>transverse steel ratio, (\rho_w) (%)</td>
<td>0-1.18</td>
<td>0.24</td>
</tr>
<tr>
<td>total vertical steel ratio, (\rho_{tot}) (%)</td>
<td>0.815-7.6</td>
<td>2.08</td>
</tr>
<tr>
<td>geometric ratio of FRP, (\rho_f) (%)</td>
<td>0.01-5.31</td>
<td>0.605</td>
</tr>
<tr>
<td>nominal FRP strength (MPa)</td>
<td>113-4830</td>
<td>2755</td>
</tr>
<tr>
<td>elastic modulus of FRP, (E_f) (GPa)</td>
<td>5.8-390</td>
<td>166</td>
</tr>
<tr>
<td>lapping-to-bar-diameter ratio, (l_o/d_b)</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>
FRP-wrapping of plastic hinges in rectangular members with continuous bars per EN 1998-3:2005

- $M_R, M_y$: Enhanced by FRP jacket (by 9% w.r.to calculated w/o confinement)
- Effective (elastic) stiffness $EI_{\text{eff}}$: unaffected by FRP; pre-damage: 35% drop
- EN1998-3:2005: Flexure-controlled ultimate chord rotation, $\theta_u$:
  - Confinement by FRP increases that due to the stirrups by $\alpha_f \rho_f f_{f,e} / f_c$, where:
    - $\rho_f = 2t_f / b_w$: FRP ratio // direction of loading;
    - $f_{f,e}$: FRP effective strength:
      \[
      f_{f,e} = \min \left( f_{u,f}, \varepsilon_{u,f} E_f \right) \min \left( 0.5, 1 - 0.7 \frac{\min \left( f_{u,f}, \varepsilon_{u,f} E_f \right) \rho_f}{f_c} \right)
      \]
    - $f_{u,f}, E_f$: FRP tensile strength & Modulus;
    - $\varepsilon_{u,f}$: FRP limit strain. CFRP/AFRP: $\varepsilon_{u,f} = 1.5\%$; GFRP: $\varepsilon_{u,f} = 2\%$
    - FRP-confinement effectiveness:
      \[
      \alpha_f = 1 - \frac{(h - 2R)^2 + (b - 2R)^2}{3bh}
      \]
    - $b, h$: sides of section;
    - $R$: radius at section corner
Test-model comparison - yield properties for FRP-wrapping of rectangular columns with continuous bars per EN 1998-3:2005

no. tests: 203 (pre-damaged or not)
median = 1.09, CoV = 19.4%

no. tests: 159 (no pre-damage)
median = 1.03, CoV = 37.3%

\[ M_{y,\text{exp}} = 1.09 M_{y,\text{pred}} \]

\[ \theta_{y,\text{exp}} \% \]

\[ E_{\text{Ieff}} \text{ (no pre-damage), no. tests: 159, median = 1.02, CoV = 30\%} \]
Yield properties for FRP-wrapping of the plastic hinge and continuous bars – Biskinis & Fardis 2007, 2013

• Yield moment, $M_y$:
  - Strength of FRP-confined concrete $f_{cc}$, instead of $f_c$, in section-analysis for $\phi_y, M_y$, including the calculation of the concrete Elastic Modulus, $E_c$;
  - $E_c$ may be estimated per fibMC2010, using $f_{cc}$ instead of $f_c$:
    $$E_c = 10000(f_{cc}(\text{MPa}))^{1/3}$$
  - $f_{cc}$ from (widely used) Lam & Teng 2003:
    $$f_{u,f} = E_f(k_{eff}\varepsilon_{u,f})$$
    $f_{u,f}$: effective strength of FRP, $E_f$: Elastic Modulus of FRP, $\varepsilon_{u,f}$: FRP failure strain, $k_{eff}$: FRP effectiveness factor, equal to 0.6 per Lam & Teng

• Chord rotation at yielding, $\theta_y$, effective stiffness, $EI_{eff}$:
  - Yield curvature $\phi_y$ calculated with $f_{cc}$ (as above) & multiplied times correction factor 1.06
  - $EI_{eff} = M_y L_s/3\theta_y$
Test-model comparison - yield properties for FRP-wrapping of rect. columns with continuous bars per Biskinis & Fardis 2007, 2013

no. tests: 203 (pre-damaged or not) median=1.06, CoV=19.3%

no. tests: 159 (no pre-damage) median=1.01, CoV=37.3%

EI_{eff} (no pre-damage): no. tests: 159, median=0.99, CoV=30.4%

- Pre-damaged or not:

\[
\theta_{u}^{pl} = 0.0185 \cdot (1 - 0.52a_{cy}) \left(1 + \frac{a_{sl}}{1.6}\right)(0.25)^{v} \left(\frac{\text{max}(0.01, \omega')}{\text{max}(0.01, \omega)}\right)^{0.3} f_{c}^{0.2} \left(\frac{L_{s}}{h}\right)^{0.35} \left[\frac{ap_{w}f_{y,c}}{f_{c}} + \frac{ap_{f}u}{f_{c}}\right]^{0.35} 1.275^{100 \rho_{d}}
\]

with:

\[
\left(\frac{ap_{f}u}{f_{c}}\right)_{f,\text{eff}} = a_{f}c_{f}\min\left[0.4; \frac{\rho_{f}f_{u,f}}{f_{c}}\right] \left[1 - 0.5\min\left[0.4; \frac{\rho_{f}f_{u,f}}{f_{c}}\right]\right]
\]

c_{f} = 1.8 for CFRP or polyacetal fiber (PAF) sheets,
c_{f} = 0.8 for GFRP or AFRP.

\(f_{u,f} = E_{f}(k, \varepsilon_{u,f})\): effective FRP strength
Test-model comparison – ultimate cyclic chord rotation for FRP-wrapping of rect. columns with continuous bars: no. tests 128 (pre-damaged or not)

EN 1998-3:2005
median=1.09, CoV=30.6%

Biskinis & Fardis 2013
median=1.025, CoV=30.4%
Alternative ultimate cyclic chord rotation for continuous bars & FRP-wrapping – per GCSI (KANEPE)

\[ \frac{f_{cc}}{f_c} = 1.125 + 1.25 \alpha \omega_{wd} \]

\( \omega_{wd} \): FRP volumetric ratio
\( \alpha \): FRP-confinement effectiveness factor

CFRP (adopted here also for AFRP and PAF):

\[ \varepsilon_{cu} = 0.0035 \left( \frac{f_{cc}}{f_c} \right)^2 \]

GFRP:

\[ \varepsilon_{cu} = 0.007 \left( \frac{f_{cc}}{f_c} \right)^2 \]

FRP effective strength:

\[ f_{f,e} = f_{u,j} \psi \]

\( \psi \): reduction factor for number of layers (k);
\( \psi = 1 \) for \( k < 4 \)
\( \psi = k^{-1/4} \) for \( k \geq 4 \)

\[ \mu_\theta = \frac{\theta_u}{\theta_y} \approx \mu_\phi = \left( \mu_\phi + 2 \right)/3 \]

Test-to-predicted ultimate cyclic chord rotation, \( \theta_u \) – no. tests: 128, median=1.75, CoV=54.5%
FRP-wrapped rectangular members with **ribbed bars** lap-spliced over length $l_o$ in the plastic hinge: EN 1998-3:2005 & other options

**EN 1998-3:2005:**

1. Both bars in pair of lapped compression bars count as compression steel.
2. For the yield properties ($M_y$, $\phi_y$, $\theta_y$), the stress $f_s$ of tension bars is:

   $$ f_s = f_y \left( \frac{l_o}{l_{oy,min}} \right), \quad \text{if } l_o < l_{oy,min} = \left(0.2f_y/\sqrt{f_c}\right) d_b \quad (f_y, f_c \text{ in MPa}) $$

   \[ l_{oy,min} \text{: one-third shorter than without FRP.} \]

3. Ultimate chord rotation

   $$ \theta_u = \theta_y + \theta_{pl} \left( \frac{l_o}{l_{ou,min}} \right), \quad \text{if } l_o < l_{ou,min} = d_b f_y \left( \frac{1.05 + 14.5 \alpha_i \rho_f f_{f,e}}{\rho_f f_{f,e}} \right) \sqrt{f_c} $$

   \[- f_c: \text{MPa}, \quad \rho_f = 2t_f/b_w: \text{FRP ratio} // \text{loading}, \quad f_{f,e}: \text{effect. FRP strength (MPa)}, \]

   \[- \alpha_i = \alpha_f (4/n_{tot}) \quad (n_{tot}: \text{total lap-spliced bars}; \text{only the 4 corner bars restrained}). \]

**Or, Eligehausen & Lettow 2007 for fibMC2010:**

$$ f_s = 51.2 \left( \frac{l_b}{d_b} \right)^{0.55} \left( \frac{f_c}{20} \right)^{0.25} \left( \frac{20}{\max(d_b; 20mm)} \right)^{0.2} \left[ \left( \frac{c_d}{d_b} \right)^{1/3} \left( \frac{c_{max}}{c_d} \right)^{0.1} \right] \leq f_y, \quad kK_{tr} = \frac{1}{n_b d_b} \left( \frac{k_s n_l A_{sh}}{s_h} + \frac{k_f n f_t f E_f}{E_s} \right) $$

- $c_d = \min[a/2; c_1; c] \geq d_b$, $c_d \leq 3d_b$
- $c_{max} = \max[a/2; c_1; c] \leq 5d_b$
- $f_y, f_c \text{ in MPa}$

---

**Example:**

- $a = 3, c \Rightarrow n_l = 1, n_s = 1, k = 5$
- $a = 3, c \Rightarrow k = 0$
- $n = 2, n_s = 3, k = 10$
Test-model comparison - Yield properties for FRP-wrapping of rect. columns with bars lap-spliced over length $l_o$ per EN1998-3:2005, no. tests 45

$M_y$: median = 1.075, CoV = 10.5%

$\theta_y$: median = 1.075, CoV = 17.7%

- no. tests: 42
  - median = 1.00
  - CoV = 15.2%

- no. tests: 42
  - median = 0.98
  - CoV = 22%

\[ \theta_{y,exp} = 0.98 \theta_{y,pred} \]

\[ (M/L_3 \theta_y)_{exp} = 0.96 (M/L_3 \theta_y)_{pred} \]
Extension of empirical ultimate plastic chord rotation for rectangular members with lap-spliced bars & FRP-wrapping – Biskinis & Fardis 2007, 2013

- Required lapping for no adverse effect of lap-splice on ultimate deformation

\[ l_{ou,\text{min}} = \frac{d_b f_y}{1.05 + 14.5 \left( \frac{2}{n_{\text{tension}}} \right)^2 \left( \frac{ap f_u}{f_c} \right)_{f,\text{eff}}} \sqrt{f_c} \]

with:

\[ \left( \frac{ap f_u}{f_c} \right)_{f,\text{eff}} = a_f c_f \min \left[ 0.4; \frac{\rho f u, f}{f_c} \right] \left( 1 - 0.5 \min \left[ 0.4; \frac{\rho f f u, f}{f_c} \right] \right) \]

\[ f_{u,f} = E_f (k_{\text{eff}} \varepsilon_{u,f}) \]

\[ n_{\text{tension}}: \text{number of lapped bars on tension side of section} \]

- If \( l_0 < l_{ou,\text{min}} \), \( \theta_u^{pl} \) is reduced as:

\[ \theta_u^{pl} = \min \left( 1, \frac{l_o}{l_{ou,\text{min}}} \right) \theta_u^{pl} \]

\( (ap f_u/f_c)_{f,\text{eff}} \) neglected in \( \theta_u^{pl} \) if \( 2/n_{\text{tension}} \leq 0.5 \)
Test-model comparison – ultimate cyclic chord rotation of FRP-wrapped rectangular columns w/ lap-spliced bars, no. tests 44 (pre-damaged or not)


EN 1998-3:2005
median=0.89, CoV=36.3%

Biskinis & Fardis 2013
median=1.005, CoV=23.2%

Median: \( \theta_{u,\text{exp}} = 0.885 \theta_{u,\text{pred}} \)

Median: \( \theta_{u,\text{exp}} = 1.005 \theta_{u,\text{pred}} \)
Extension of ultimate plastic chord rotation model with curvatures and plastic hinge length to circular columns with lap-spliced bars & FRP-wrapping – Biskinis & Fardis 2013

\[ \theta_u = \theta_y + (\varphi_u - \varphi_y)L_{pl}\left(1 - 0.5\frac{L_{pl}}{L_s}\right) + a_{sl}\Delta \theta_{u,\text{slip}} \]

\[ \Delta \theta_{u,\text{slip}} = 5.5d_{bl}\varphi_u \]

\( L_{pl}, f_{cc}, \varepsilon_{cu} \) as in members (FRP-wrapped or not) with continuous bars

Steel strain at ultimate deformation of member depends on lap length, if \( l_o < l_{ou,\text{min}} \):

\[ \varepsilon_{su,l} = \left[1.2 \frac{l_o}{l_{ou,\text{min}}} - 0.2\right] \varepsilon_{su} \geq \frac{l_o}{l_{oy,\text{min}}} \frac{f_y}{E_s} \]

with:

\[ l_{ou,\text{min}} = \frac{d_{bl}f_y}{\left(5/6 + 6\alpha \rho_{sh} f_{yh}/f_c\right)\sqrt{f_c}} \]

Test-to-prediction ratio, no. tests: 37
median = 0.97, CoV = 38.6%
Cyclic shear resistance of FRP-wrapped rectangular members as controlled by diagonal tension, per EN1998-3:2005

\[ V_R = \frac{h - x}{2L_s} \min \left( N, 0.55A_c f_c \right) + \left( 1 - 0.05 \min \left( 5, \mu_p^l \right) \right) \left[ 0.16 \max \left( 0.5, 100 \rho_{tot} \right) \left( 1 - 0.16 \min \left( 5, \frac{L_c}{h} \right) \right) \sqrt{f_c A_c + V_w} \right] \]

- \( V_f = \min(\epsilon_{u,f} E_{u,f}, f_{u,f}) \rho_f b_w z/2 \) : FRP-contribution to cyclic shear resistance:
  - \( \rho_f \): FRP ratio, \( \rho_f = \frac{2t_f}{b_w} \);
  - \( f_{u,f} \): FRP tensile strength.

- \( V_w = \rho_w b_w z f_{yw} \): contribution of web steel (\( b_w \): web width, \( z \): int. lever arm; \( \rho_w \): steel ratio)

- \( \rho_{tot} \): total longitudinal steel ratio

- \( h \): section depth

- \( x \): depth of compression zone

- \( A_c = b_w d \)

Test-to-prediction ratio vs \( \mu \), no. tests 12, median=0.99, CoV=14.8%:

- In a FRP-retrofitted member the shear resistance as controlled by diagonal tension cannot exceed the shear resistance of old member as controlled by web crushing.
RC jackets per EN1998-3:2005
Concrete Jackets (continued/anchored in joint; w/ or w/o lap splices in old member)

Calculation assumptions:

- Full composite action of jacket & old concrete assumed (jacketed member: monolithic”), even for minimal shear connection at interface (roughened interface, steel dowels epoxied into old concrete: useful but not essential);
- $f_c$ of “monolithic member” = that of the jacket (avoid large differences in old & new $f_c$);
- Axial load considered to act on full, composite section;
- Longitudinal reinforcement of jacketed column: mainly that of the jacket. Vertical bars of old column considered at actual location between tension & compression bars of composite member (~ “web” longitudinal reinforcement), with its own $f_y$;
- Only the transverse reinforcement of the jacket is considered for confinement;
- For shear resistance, the old transverse reinforcement taken into account only in walls, if anchored in the (new) boundary elements;
- The detailing & any lap-splicing of jacket reinforcement are taken into account.

Then:

- $M_R$ & $M_y$ of jacketed member: ~100% of those of “monolithic member” calculated w/ assumptions above.
- $\theta_y$ of jacketed member for pre-yield (elastic) stiffness: ~105% of those of “monolithic member” calculated w/ assumptions above.
- Shear resistance of jacketed member: ~100% of those of “monolithic member” calculated w/ assumptions above.
- Flexure-controlled ultimate deformation $\theta_u$: ~100% of those of “monolithic member” calculated w/ assumptions above.

If jacket bars are not continued/anchored in the joint:

The jacket is considered only to confine fully the old column section.
54 jacketed members w/ or w/o lap splices: test-to-calculated as monolithic

- Continuous bars
- Plain 15db laps
- Deformed 15db laps
- Plain 25db laps
- Deformed 30db laps
- Deformed 45db laps
- Non-anchored jacket bars
- Group average
- St. dev. of group mean

- $M_y, \text{calc}$
- $M_y, \text{exp} / M_y, \text{calc}$
- $\theta_y, \text{exp} / \theta_y, \text{calc}$
- $\theta_u, \text{exp} / (\theta_u + \theta_u, \text{calc})$

- $EI_{eff}$
- $EI_{exp} / EI_{eff}$

Steel jackets per EN1998-3:2005
Steel Jackets (not continued/anchored in joint)

Jacket stops before the joint (several mm gap to joint face)

- Flexural resistance, pre-yield (elastic) stiffness & flexure-controlled ultimate deformation of RC member is not enhanced by jacket (flexural deformation capacity ~ same as in old member inside jacket, no benefit from confinement);

- 50% of shear resistance of steel jacket, $V_j = A_{jyj}h$, can be relied upon for shear resistance of retrofitted member (suppression of shear failure before or after flexural yielding);

- Lap-splice clamping via friction mechanism at jacket-member interface, if jacket extends to ~ 1.5 times splice length and is bolt-anchored to member at end of splice region & ~ 1/3 its height from the joint face (anchor bolts at third-point of side)